

# AE-9/AP-9 Radiation belt models (An introduction for scientists)

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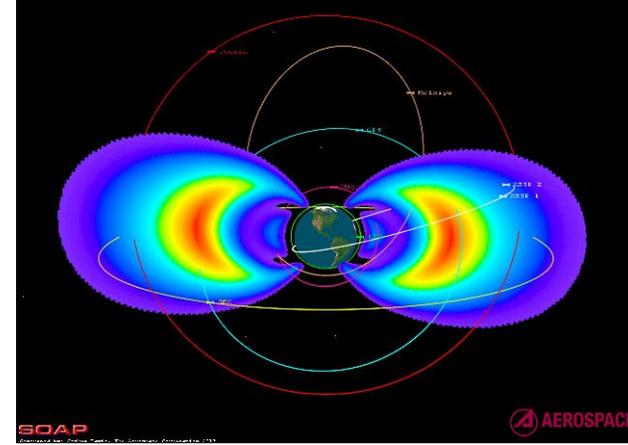
Note: This slide pack borrows heavily from materials developed by the AE9/AP9 team, including contributions from AFRL and AER.

# Overview

- What is AE9/AP9?
- Model components
- Model internal representation (spatial)
- Model internal representation (statistical)
- Statistical dynamics
- Data sources
- Scientific applications

# What is AE9/AP9?

- A climatology model built for satellite design
  - *1 keV to 10 MeV trapped electrons*
  - *1 keV to 2 GeV trapped protons*
  - *1 keV to 164 keV trapped helium, oxygen ions*
- Statistically describes average and transient dynamics
  - *User provides times/locations/look-directions of interest or orbit elements*
  - *Model provides electron, proton, ion flux vs energy along trajectory/orbit*
- Model is re-run multiple times for user's different statistical needs
  - *Mean or static percentile*
  - *Perturbed mean scenarios (each scenario includes realistic error on the mean)*
  - *Monte Carlo scenarios (adds dynamics for quantifying transient effects)*
- Written in C++, source code available upon request
  - *GUI for convenience*
  - *Command line utility for scripting*
  - *API for programmatic interface from other applications/languages*
- Get it at <https://www.vdl.afrl.af.mil/programs/ae9ap9/>
- Slowly renaming it IRENE – International Radiation Environment Near Earth



# Model Components

Component	Species	Energy Range
AE9	Electrons	40 keV – 10 MeV
SPME	Electrons	1 – 40 keV
AP9	Protons	100 keV to 2 GeV
SPMH	Protons (Hydrogen)	1 – 164 keV
SPMHE	Helium (He+)	1 – 164 keV
SPMO	Oxygen (O+)	1 – 164 keV

- AE – Aerospace Electron,
- AP – Aerospace Proton
- SPM – Standard Plasma Model
- Currently, these components are run separately
- Integral plasma fluxes involving SPMH and SPME are computed by “stitching together” AE9+SPME or AP9+SPMH using an IntegralPlasma utility
- In a future version, AE9+SPME and AP9+SPMH will be combined into a unified energy grid
- SPME and SPMO are rarely used

# Internal Representation (Spatial)

- All model components represent differential number flux spectrum on a 2-d spatial grid:  $j(E, Q_2, Q_3)$  in units of  $\#/cm^2/s/sr/MeV$
- For AE9 and AP9 there are two sub-grids:
  - $(Q_2, Q_3) = (K, \Phi)$ , the high altitude grid for global structure
  - $(Q_2, Q_3) = (K, h_{min})$ , the low altitude grid for steep altitude gradients
  - Invariant around drift orbit
  - $K$  – Kaufmann’s  $K = I\sqrt{B_m}$ , modified second invariant
  - $\Phi$  – Magnetic flux enclosed by drift shell (related to  $L^*$ )
  - $h_{min}$  – Minimum mirror altitude on field line
- For all SPM components, there is one grid:
  - $(Q_2, Q_3) = (\alpha_{eq}, L_m)$
  - Bounce invariants treated as drift invariants
  - $\alpha_{eq}$  equatorial pitch angle
  - $L_m$  McIlwain  $L$
  - No dependence on local time yet
- Uses IGRF + Olsen-Pfitzer Quiet (OPQ) magnetic field model
- Employs Pfitzer’s “Fast IGRF” that truncates IGRF moments based on location to avoid computing unneeded spherical harmonics
- $\Phi, h_{min}$  computed via neural networks fit to precomputed drift traces

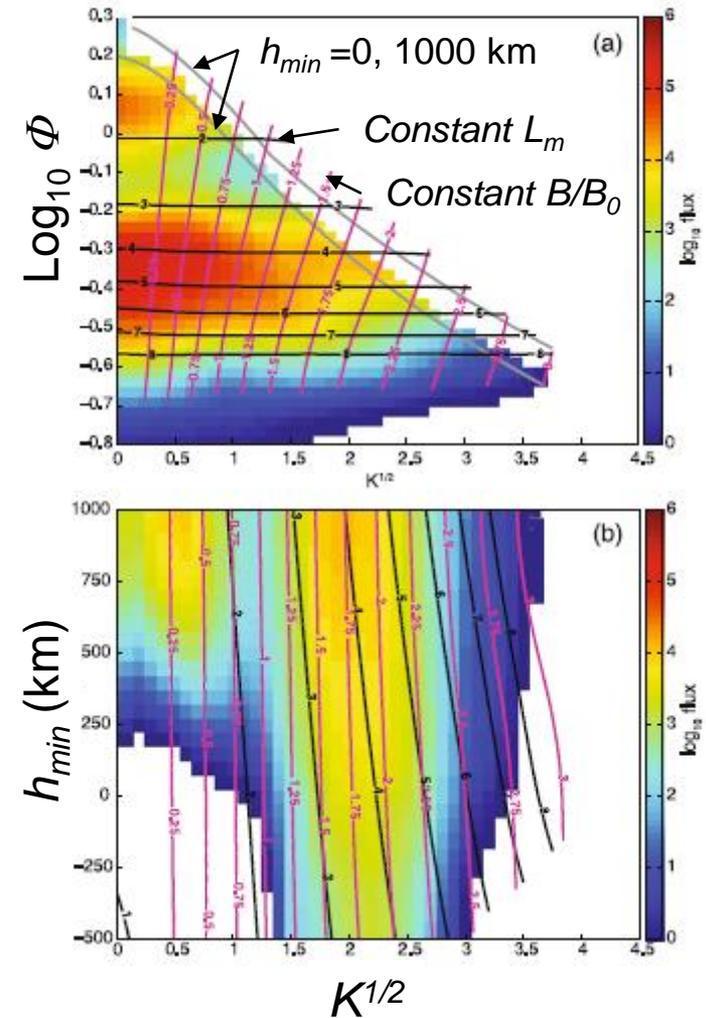


Figure courtesy of AFRL

# Internal Representation (Statistical)

- At each grid point the model assumes either a log-normal or Weibull statistical distribution:
  - *AP9 and SPM use log-normal, AE9 uses Weibull*
  - *Log-normal:  $f(x) = \frac{\exp[-(\ln x - \mu)^2 / 2\sigma^2]}{x\sigma\sqrt{2\pi}}$*
  - *Weibull:  $f(x) = \gamma(x/x_0)^{\gamma-1} \exp[-(x/x_0)^\gamma] / x_0$*
- The model captures the two parameters of these distributions indirectly:
  - *$\theta_1 = \text{natural log of median flux (x = flux)}$*
  - *$\theta_2 = \text{natural log of difference between 95}^{\text{th}} \text{ percentile flux and median}$*
  - *Ensures 95<sup>th</sup> percentile > median for all real values of  $(\theta_1, \theta_2)$*
- Model stores  $(\theta_1, \theta_2)$  at each grid point and derives  $(\mu, \sigma)$  or  $(x_0, \gamma)$  at run time
- Model also has  $\underline{\underline{S}}_\theta$ , such that  $\underline{\underline{S}}_\theta \underline{\underline{S}}_\theta^T$  is a matrix of correlated errors for  $(\theta_1, \theta_2)$  over the entire grid.
  - *$\underline{\underline{S}}_\theta$  mainly represents error associated with different choices of data sets used to derive  $\theta$ , but also includes sensor/coordinate binning errors*
  - *$\underline{\underline{S}}_\theta$  is used to perturb all  $(\theta_1, \theta_2)$  for perturbed mean and Monte Carlo states*
- Given a global map of  $(\theta_1, \theta_2)$  (perturbed or not), one can compute any percentile or statistical moment (usually the mean) at all grid points.

# Statistical Dynamics

- Dynamics\* are represented as a multi-lag autoregressive process:

$$-\vec{q}_t = \sum_{i=1}^{N_G} \underline{\underline{G}}_i \vec{q}_{t-\tau_i} + \underline{\underline{C}} \vec{\eta}_t$$

- State vector  $\vec{q}_t$  is a set of principal component amplitudes

- $\underline{\underline{G}}_i$  control persistence at various timescales  $\tau_i$

- $\underline{\underline{C}}$  controls innovation

- $\vec{\eta}_t$  is Gaussian white noise with zero mean, unit variance

- Principal components given by  $\underline{\underline{Q}}$ :

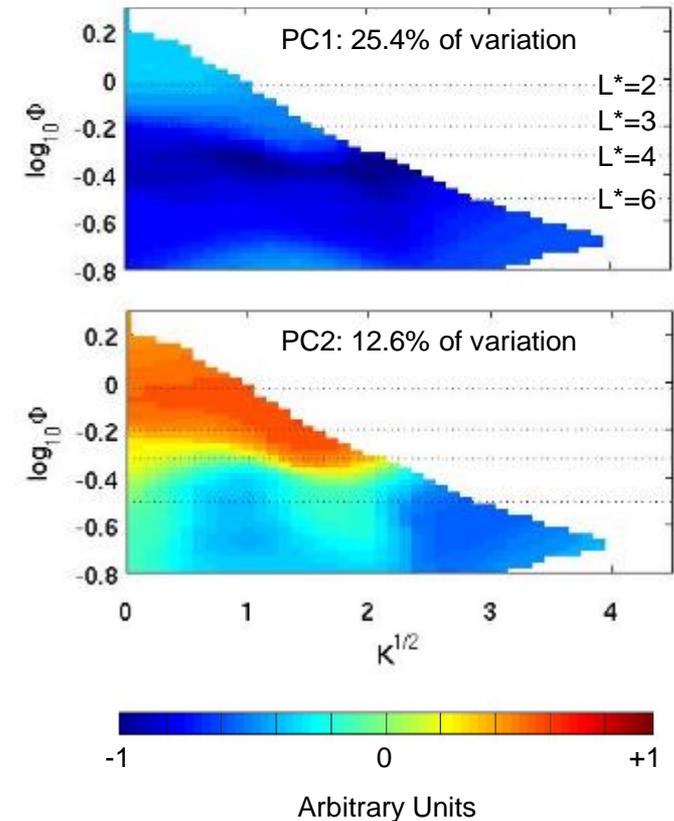
- $\underline{\underline{\Sigma}} = \underline{\underline{Q}} \underline{\underline{Q}}^T =$  spatial covariance matrix on model grid

- $\underline{\underline{Z}}_t = \underline{\underline{Q}} \vec{q}_t =$  Gaussianized flux variables

- $x_t^{(i)} = F^{-1} \left[ \Phi \left( z_t^{(i)} \right); \theta_1^{(i)}, \theta_2^{(i)} \right]$

- $[\Phi(z)$  is cumulative Gaussian distribution,  $F$  is cumulative log-normal or Weibull distribution]

AE9 Principal Components at 750 keV



\*Dynamics are only available for AE9 and AP9, not SPM models

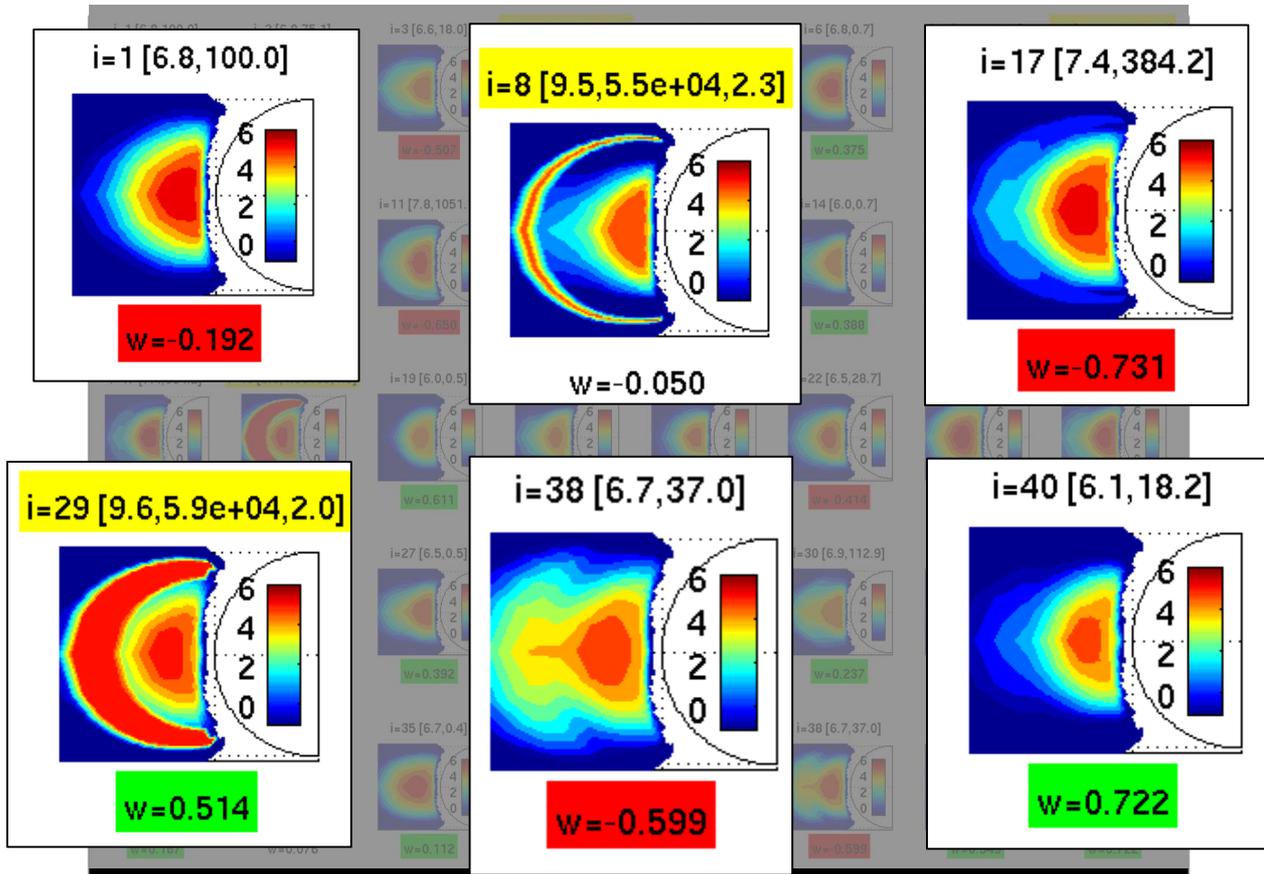
# Data Sources

Protons	Orbit				Energy [MeV]																									
	LEO	MEO	HEO	GEO	0.10	0.20	0.40	0.60	0.80	1.00	2.00	4.00	6.00	8.00	10.0	15.0	20.0	30.0	50.0	60.0	80.0	100.0	150.0	200.0	300.0	400.0	700.0	1200.0	2000.0	
AP9																														
CRRES/PROTEL																														
S3-3/Telescope																														
ICO/Dosimeter																														
HEO-F3/Dosimeter																														
HEO-F1/Dosimeter																														
TSX5/CEASE																														
POLAR/IPS																														
POLAR/HISTp																														
TacSat-4/CEASE																														

Electrons	Orbit				Energy [MeV]																									
	LEO	MEO	HEO	GEO	0.04	0.07	0.10	0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00	8.50	10.0					
AE9																														
CRRES/MEA/HEEF																														
ICO/Dosimeter																														
HEO-F3/Dos/Tel																														
HEO-F1/Dos/Tel																														
TSX5/CEASE																														
POLAR/HISTe																														
POLAR/IES																														
GPS/BDDII																														
LANL GEO/SOPA																														
SAMPEX/PET																														
SCATHA/SC3																														

Plasma	Orbit				Energy [keV]											
	LEO	MEO	HEO	GEO	0.50	1.00	2.00	4.00	6.00	12.00	20.0	40.0	60.0	80.0	100.0	150.0
SPM																
POLAR/CAMMICE/MICS																
POLAR/HYDRA																
LANL GEO/MPA																
THEMIS/ESA																

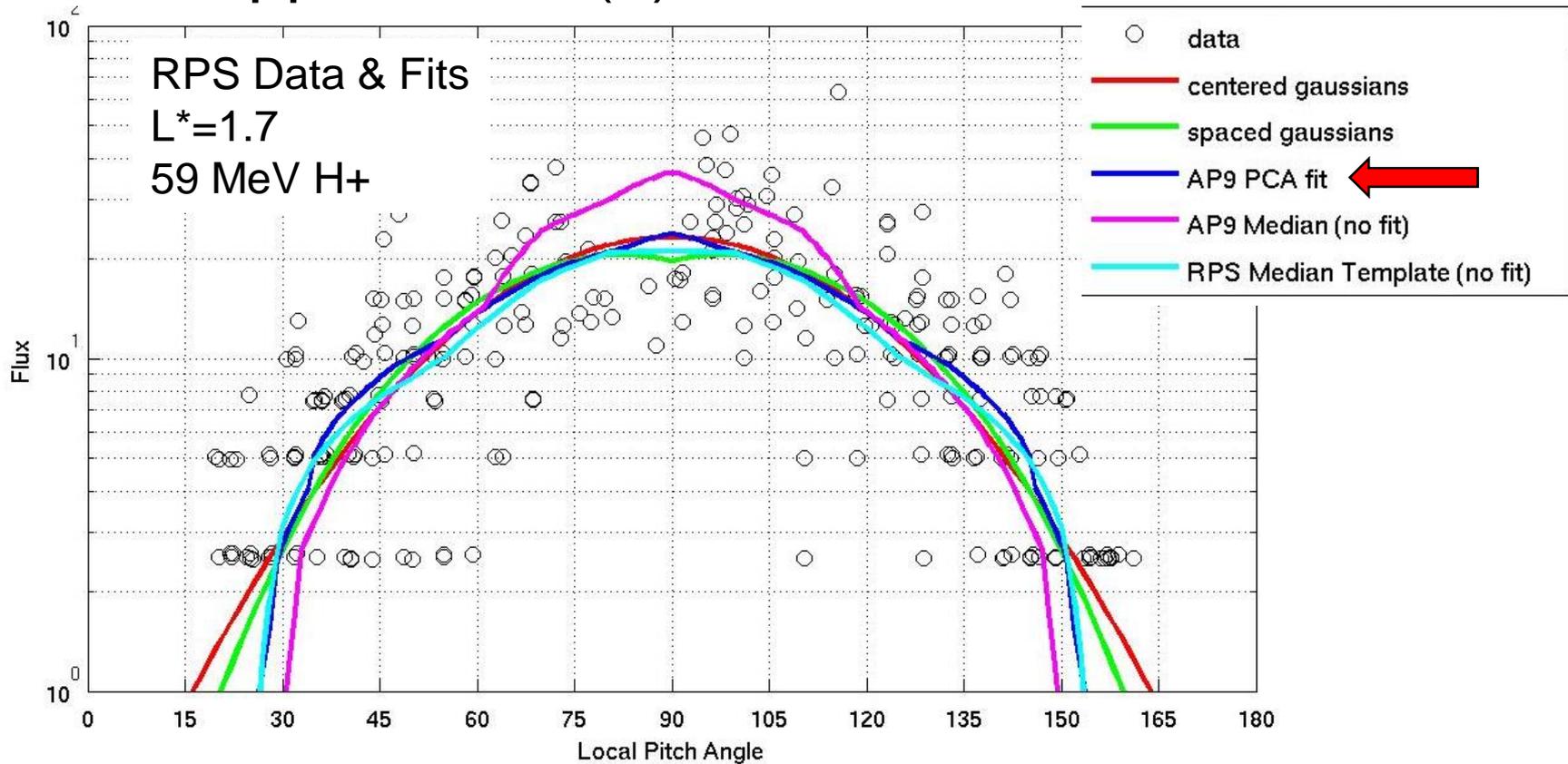
# Scientific Applications (1) Initial and Boundary Conditions



- AP9 Monte Carlo scenario states were used to populate an ensemble of initial conditions for a proton belt data assimilation mode
- Selesnick Inner Zone Model
- Data Assimilation by Tim Guild
- Can also be used to supply multiple, realistic boundary conditions for global simulations

Figure courtesy of Tim Guild, Aerospace

# Scientific Applications (2) Priors



- AP9 provides priors for angular inversion of Relativistic Proton Spectrometer (RPS) data on Van Allen Probes
- Can likewise be used for energy spectrum inversion
- Could also be used as “background” model for 3Dvar data assimilation w/o a physical model

# Summary

- AE9/AP9 is an empirical, statistical climatology model meant to support satellite design
- It covers a wide range of energies from hot plasma to radiation belts
- It uses physics-based coordinates to organize the data
- It includes statistical models of local variation, global dynamics, and model uncertainty
  
- It can be used to supply initial or boundary conditions for physical simulations
- It can be used to provide priors for sensor data inversion and data assimilation
  
- AE9/AP9 is being gradually renamed to IRENE to recognize the growing international contributions

# Abstract

The presentation introduces AE9/AP9 and describes some potential uses in scientific studies.

AE9/AP9 is a model of the trapped particle fluxes from 1 keV to 2 GeV. It is a statistical model derived from in situ observations. AE9 is the electron model from 40 keV to 10 MeV, and AP9 is the proton model from 100 keV to 2 GeV. The AE9 and AP9 models use a combination of low-altitude and high altitude drift invariant grids to capture the structure of the radiation belts. The SPM models use high altitude coordinate system only. These two models can provide static mean and percentile environments as well as dynamic Monte Carlo time series environments. The dynamic scenario states are evolved in time via a statistical auto-regressive process rather than solving physical dynamic equations. A separate set of “standard plasma models” (SPM) cover the plasma energies down to 1 keV for electrons, protons, oxygen, and helium. The SPM family of models can provide static mean and percentile environments, but does not yet include dynamics.

One scientific application of the AE9/AP9 family of models is generation of realistic initial and boundary conditions. The models can also be used as Bayesian priors for sensor data inversion or data assimilation.