Probing the Upper Atmosphere and Ionosphere with Large Radars

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Target audience:

Bright graduate students who don't know much about radars but have inquiring minds

Goal of this talk:

To give you some idea of

1. What radars are good for

2. How they differ from lidars (the uses of phase coherence)

3. Some basic radar concepts and techniques
More specifically, we will try to cover (pretty fast), or at least mention

1. The radar equation

2. The difference between scattering from hard and **SOFT** (main emphasis) targets

3. Some properties of soft targets; e.g.,
   - Range dependence of the scattered signal strength
   - The Bragg condition – why the radar picks out a single spatial Fourier component of the refractive index fluctuations in a random medium
   - Over- vs under-spread and why it matters; range and frequency aliasing
   - Statistical ideas – why one sample isn't enough even if the signal-to-noise ratio S/N is very large
4. Some radar techniques; e.g.,

- FFT analysis of the Doppler spectrum from under-spread targets (Easy to do. Can measure very small Doppler shifts, for example 1 Hz, even if the pulse bandwidth is 1 MHz.)

- ACF analysis of the spectrum from overspread targets (Not so easy. The price of beating the Fourier uncertainty principle is the addition of radar "clutter", or signals from unwanted ranges that act like noise.)

- Pulse compression (How to turn a long low power pulse into a short high power one with the same number of joules.)

- Radar interferometry (How to locate strong scatterers precisely within the scattering volume.)

5. Incoherent scatter

The very weak scatter from purely thermal fluctuations (the irreducible minimum level) in plasma density. For a plasma in thermal equilibrium, the scattered power and signal Doppler spectrum depend in a quantitatively known way on the plasma density, temperatures, ion composition, drift velocity, etc.

So by measuring the power spectrum, or more likely the signal autocorrelation function (ACF), we can determine most of the important plasma parameters via least squares fitting to the theory.
Radar Equation

\[ P_{\text{rec}} = \left( \frac{P_t G_t}{4\pi R_1^2} \right) \left( \frac{A_{\text{rec}}}{4\pi R_2^2} \right) \]

incident power density \hspace{1cm} \text{total scattering X-section}

fraction received

For a single transmit-receive antenna (pulsed radar)

\[ P_{\text{rec}} = P_t \frac{G A_{\text{eff}}}{(4\pi R^2)^2} \Sigma \]

\[ G = \frac{4\pi A_{\text{eff}}}{\lambda^2} \quad \text{(Antenna Theory)} \]

So

\[ P_{\text{rec}} \sim P_t \frac{\lambda^2 G^2}{R^4} \Sigma \sim P_t \frac{A_{\text{eff}}^2}{\lambda^2 R^4} \Sigma \]

What is Σ? [Soft vs Hard Target]

What is G or A_{\text{eff}}? [Near Field vs Far Field]
HARD TARGET

DOES NOT FILL BEAM - e.g. PLANE, MISSILE, SATELLITE

Σ INDEPENDENT OF RANGE

HENCE

\[ P_{\text{rec}} \sim R^{-4} G^2 \]

SOFT TARGET

FILLS BEAM - e.g. SCATTER FROM ATMOSPHERE, IONOSPHERE

\[ \Sigma = \sigma \text{ per unit volume} \]

AND

\[ V_s = \Omega R^2 l = \frac{4\pi}{G} R^2 l \]

WHERE

\[ l = \text{MIN [PULSE LENGTH = CT/2, LAYER THICKNESS]} \]

HENCE

\[ P_{\text{rec}} \sim \frac{\lambda^2 G \sigma l}{R^2} \sim \frac{\sigma l A_{\text{eff}}}{R^2} \]
What is a "soft" target?

\[ \frac{P_r}{P_t} = \left( \frac{G_e}{4\pi r^2} \right) \sigma_{\text{target}} \left( \frac{A_{\text{eff}}}{4\pi r^2} \right) \sim \frac{G_e A_{\text{eff}}}{r^4} \sigma_{\text{target}} \]

But \( \sigma_{\text{target}} \sim (\text{beam area})(\text{pulse length}) \sim \frac{r^2}{G_e} \text{ pulse} \)

\[ \Rightarrow \frac{P_r}{P_t} \sim \frac{A_{\text{eff}} r^2}{r^2} \]

(Born \( \frac{A_{\text{eff}}}{\lambda^2 r^4} \) for hard target)

Born approximation always valid for cases of interest

E.g. single (weak) scattering only,

scatter doesn't alter (attenuate) incident beam

Goal: Determine the statistical properties of
the scattering medium.

Method: Measure the statistical properties (power spectrum
or auto-correlation function) of the
scattered signal, which is a Gaussian random variable.
Signal statistics vs medium properties

Straightforward to show that (for a plasma)

\[ \langle E_3(t) E_3^*(t+\tau) \rangle \sim \langle \Delta N(k, t) \Delta N^*(k, t+\tau) \rangle \]

Signal ACF

\[ \iint \langle \Delta N(r, t) \Delta N(r', t+\tau) \rangle e^{i k \cdot r'} \, d^3 r' \]

\( N = \text{electron density} \)

\( k = k_{\text{incident}} - k_{\text{scattered}} \)

\( \rightarrow 2k_{\text{inc}} \) for backscatter

(Bragg condition)

Fourier transforming in time gives

\[ \langle |E_3(\omega+\omega')|^2 \rangle \sim \langle |\Delta N(k, \omega)|^2 \rangle \]

Power spectrum

Power spectrum of received signal

\( \omega = \text{Doppler shift} \)

\( \text{of electron density "waves"} \)

(Radar selects a specific \( k \), a single spatial Fourier component)

Often written as

\[ \sigma(k, \omega) \sim \langle |\Delta N(k, \omega)|^2 \rangle \]

Similarly, for fluctuations in the neutral atmosphere due to CAT etc

\[ \sigma(k, \omega) \sim \langle |\Delta \xi(k, \omega)|^2 \rangle \]

Note: <= averaging needed even if S/N >> 1
Statistics and Errors

Signal and noise are both Gaussian random variables. We have to estimate their statistical properties (e.g., mean power, power spectrum $\leftrightarrow$ ACF).

\[
\begin{align*}
\text{Signal power} & \equiv S \propto \frac{P_{\text{trans.}}}{R^2} \\
\text{(soft target)} & \quad \frac{A_{\text{aut. medium}}}{R^2} \quad \text{per unit volume}
\end{align*}
\]

\[
\text{Noise power} \equiv N = K \frac{T}{\text{Boltz system receiver}}.
\]

For "matched" filtering (gives max S/N), we often (but not always) have

\[
B_{\text{rec}} \approx \frac{1}{\tau_{\text{pulse}}}
\]

\[
\Rightarrow \frac{S}{N} \propto \tau_{\text{pulse}}^2 \propto (\Delta R)^2 \quad \text{range resolution}
\]

Using many samples of the signal, we form an estimator of the true signal power, or ACF, etc.
For example,
\[
\hat{S} + \hat{N} = \frac{1}{K} \sum_{i=1}^{K} |V_i|^2 \quad \text{transmitter on}, \quad \hat{N} = \frac{1}{K} \sum_{i=1}^{K} |V_i|^2 \quad \text{trans. off}
\]
\[
\hat{R}(\sigma) = \frac{1}{K} \sum_{i=1}^{K} V_i(\sigma) V_i^*(\sigma + \tau) \Rightarrow \rho(\tau) = \hat{R}(\tau) / \hat{R}(0)
\]

Easy to show that \( \langle \hat{N} \rangle = N \) (unbiased estimator)

Mean square errors in the estimate:
\[
\sigma_x^2 = \langle \left( \frac{\hat{X} - X}{X} \right)^2 \rangle = \frac{1}{K} \left( \frac{\hat{S} + \hat{N}}{\hat{S}} \right)^2 A \quad \text{const} \sim \Theta(1)
\]

where \( X = S, \rho(\tau), \) or whatever
\( K = \) number of independent samples used in the estimator

\( \Rightarrow \) Use tradeoffs between \( K \) and \( S/N \) wisely

\( \langle \rangle \Rightarrow \) ensemble average (= time average)
in the above
Measurement Techniques

Depends upon whether target is

- **Underspread** or **Overspread**

**Underspread**: Range and frequency aliasing both easily avoided

**Overspread**: Not so easy!

To avoid range aliasing, need \( T = \text{IPP} > \frac{2L}{c} \)

\( L = \text{target extent} \)

OK if echo is "second time around" - as long as you know this is the case (which you usually do)
To avoid frequency aliasing,

need \( PRF = \frac{1}{IPP} > \text{Doppler bandwidth} \)

So we want to satisfy 2 conditions:

1) \( IPP > \frac{2L}{c} \)

2) \( PRF > \text{Doppler BW} \)

\[ \frac{1}{PRF} < \frac{1}{\text{Doppler BW}} \]

\[
\begin{align*}
\frac{2L}{c} &< IPP < \frac{1}{\text{BW}} \\
\frac{2L}{c} &< \frac{1}{\text{Bandwidth}}
\end{align*}
\]

Yes \( \Rightarrow \) Underspread target; easy to deal with

No \( \Rightarrow \) Overspread target; harder to deal with, but can be done
**Range and Frequency Aliasing Problem**

- Need \( T > 2L/c \) (as shown) to avoid ambiguity.

- But we also need \( T < 1/2\Delta f \) to avoid frequency aliasing. (\( \Delta f = \) echo Doppler shift)
  
i.e., need \( \lvert \Delta f \rvert < 1/2T < c/4L \)

- If not true \( \Rightarrow \) target echo is aliased in frequency.
MATCHED FILTER: RECEIVER GATE WIDTH ($\sim B^{-1}$) = PULSE DURATION

AMBIGUITY: SIGNALS RECEIVED SIMULTANEOUSLY FROM $R_1$, $R_2 = R_1 + Ct/2$, ETC

RESOLUTION: $\delta R \approx \max [Ct/2, CB^{-1}_\text{rec}/2] \approx Ct/2$ FOR MATCHED FILTER

TO AVOID OVERLAPPING ECHOES, NEED $p_s(R_2) \ll p_s(R_1) \rightarrow$ LARGE $T$

BUT FOR STATISTICAL REASONS AND TO AVOID FREQUENCY ALIASING, WE WANT SMALL $T$
SAMPLING AND FREQUENCY ALIASING

\[ V(t_i, h_o) \quad V(t_i+\delta t, h_o+c\delta t/2) \]

\[ \vdots \]

\[ V(t_{i+mT}, h_o) \]

\[ \vdots \]

\[ V(t_{i+n\delta t}, h_o+nc\delta t/2) \]

LEADS TO SAMPLE ARRAY

\[ \text{EACH COLUMN CONVERTED TO FFT FOR A PARTICULAR ALTITUDE} \]

\[ \text{[V(t) IS OFTEN A COMPLEX NUMBER]} \]

\[ |\text{FFT}|^2 \]

POKER

\[ \text{[UNALIASED]} \]

\[ \text{[ALIASED]} \]

\[ T^{-1} \text{ (Hz)} \]
Underspread

Examples

- Echoes from Venus (slow rotation)
- Scatter from stratospheric and mesospheric irregularities
- Scatter from auroral irregularities (sometimes)

Simple techniques work fine, e.g.

1) Transmit evenly spaced train of pulses
2) Sample and digitize at each range of interest
3) Compute FFT for each range and average the power spectra

Also available and useful:

1) Coherent integration (this has various definitions)
2) Pulse compression using complementary code pairs (Golay codes), which have no range sidelobes

These work only if the medium correlation time \( \gg \text{IPP} \)

E.g., the stratosphere, for which

\[ \tau_{\text{correl}} \sim \text{few } \times 10^{-1} \text{ s} \]

\[ \mu \text{ IPP} \sim 1 \text{ ms perhaps} \]
Overspread

Examples:

Incoherent scatter from random thermal fluctuations in the ionosphere
Radar echoes from Mars
Radar echoes from the aurora (sometimes)

Techniques now are not so simple

Measure ACF using multipulse schemes (adds clutter)
Pulse compress with Barker (or longer) codes, but be careful about code length (must be < or ~< Ecorrel)
Combine both of the above
For mildly overspread (e.g. auroral case), can replace the usual power spectrum with double-pulse cross spectrum that unravels mild frequency aliasing

All of these could be used also for underspread targets, but usually are not

Radar interferometry is also a powerful technique for improving spatial resolution for either class of target
**PULSE COMPRESSION**

**IDEAL CASE**

\[ V(\text{envelope of } e^{i\omega t}) \]

CAN USE FREQUENCY "CHIRPING" OR PHASE CODING

IN PRACTICE USE BINARY PHASE CODING AND DECODE WITH COMPUTER OR SPECIAL PURPOSE DIGITAL OR ANALOG DEVICES

**BARKER CODES**

\[ V(\text{envelope}) \]

\[ \begin{array}{c}
  +1 \\
  0 \\
  -1
\end{array} \]

\(\tau\rightarrow\) matched filter \(\rightarrow\) decoder

\[ \phi = \text{AMBIGUITY FUNCTION (FOR NO DOPPLER)} \]

\(\phi = \text{VOLTAGE FROM SMALL STATIONARY TARGET} \]

\(\phi = \text{ACF OF CODE} \]

TARGET MUST REMAIN COHERENT FOR \(n\tau\) (UNCOMPRESSED DURATION)

GROUND CLUTTER DURATION \(\geq\) UNCOMPRESSED PULSE

MAX COMPRESSION WITH BARKER CODE (UNITY SIDELOBES) IS 15:1 \((n=13)\)

OTHER LONGER SIMILAR NON-CYCLIC CODES AVAILABLE

\(e.g., n=28 \rightarrow \text{MAX SIDELOBE OF 2}\)
Pulse Compression (con’t)

All forms of pulse compression rely on careful use of phase information.

Hence, unknown significant Doppler shifts and/or phase decorrelation (soft targets) will seriously distort/destroy the compression. (These effects are described by the full radar “ambiguity function”.)

For highly coherent, underspread targets, many other (than Barker) coding schemes are possible, some of which are very long and give very high compression ratios. e.g. complementary code pairs cyclic codes
The geometry of the electrojet interferometer. The entire Jicamarca 50-MHz array was used for transmission, but the scattered signals were received separately on the east and west quarters, whose phase centers are separated by the distance $D$ which is 208.2 m. The field-aligned "target" is assumed to occupy a small range of angles centered about the small mean angle $\theta_0$, and the subscript $\omega$ refers to the Doppler shift of the echo.

**Simplest Situation (Single Target)**

Suppose

$$V_E = V_{01} e^{i\omega t}$$
$$V_W = V_{02} e^{i\omega t - ikD \sin \theta}$$

$(k = 2\pi/\lambda$ radar$)$

Then

$$\frac{\langle |V_E V_W^*| \rangle}{\langle |V_E|^2 \rangle^{1/2} \langle |V_W|^2 \rangle^{1/2}} = e^{ikD \sin \theta} \Rightarrow \theta \text{ and } d\theta/dt \Rightarrow \text{velocity}$$

In other words, the time delay between the arrival of the signal at the E and W antennas $\Rightarrow$ a phase shift which $\Rightarrow \theta$. This idea can be extended to multiple targets with different Doppler shifts.
For point targets, this is simply 2-D direction finding.

For distributed targets, each baseline provides one point on the complex spatial ACF, which is the FT of the angular power spectrum.

In both cases, the information is provided for each Doppler shift separately.
\[|S(\omega)| \equiv e^{- (1/2)k^2D^2(\delta\theta_\omega)^2} \equiv \text{coherence} \Rightarrow \text{size}\]

\[\phi(\omega) = kD \begin{pmatrix} \sin \overline{\theta}_\omega \\ \cos \overline{\theta}_\omega \end{pmatrix} \equiv \text{phase} \Rightarrow \text{position}\]

In the auroral case the 2-D, orthogonal baseline data can be combined to give contour plots which roughly indicate the shape of the scattering center.
Measuring the ACF of an overspread target (e.g. "incoherent" scatter)

(apparently)

How do we manage to violate the Fourier Uncertainty Principle?

We use the fact that signals from disjoint scattering volumes are completely uncorrelated.

The price for this:

We add "clutter" (C), or unwanted signal from other ranges, to the noise. This clutter is averaged out, but it increases the statistical errors

\[ \delta x^2 \rightarrow \frac{1}{K} \left( \frac{S+N+C}{S} \right)^2 \]
ACF (continued)

DOUBLE OR MULTIPLE PULSE

- Sample products → \( \langle V(R_2, t) V^+(R_2, t + \tau') \rangle \rightarrow \rho(R_2, \tau') \)

- Transmit (cyclicly) different spacings and sample at all ranges to determine complete \( \rho(R, \tau') \)
  OR use multiple \( \tau, \tau' \), as we shall see

- Good range and lag resolution possible

- Clutter (echoes from unwanted ranges) adds to noise

- Can have \( \tau \neq \tau' \) and/or \( \Delta \tau \neq \delta \tau' \) (unmatched filter)
  BUT NOT RECOMMENDED (LIKELY TO GIVE SYSTEMATIC ERRORS)

- "MUST" have \( \tau \geq \Delta \tau + \delta \tau' \)

- Clutter echoes can be eliminated if pulses 1 and 2 have orthogonal polarizations (with matching receiver system)
HIGHLY DESIRABLE TO EXTEND
THIS IDEA TO
MULTIPLE PULSES

EXAMPLE: 4 PULSES AT \( t = 0, 1, 4, 6 \) PRODUCES
lags = 0, 1, 2, 3, 4, 5, 6

IN GENERAL: \( n \) PULSES \( \rightarrow \) \( n(n-1)/2 \) DIFFERENT LAGS

SOME "MISSING" LAGS FOR \( n > 4 \)

CLUTTER POWER \( \sim (n-1) \times \) SIGNAL POWER

SAME ADVICE AS IN DOUBLE PULSE CASE FOR
\( \tau \) vs \( \tau' \), \( \Delta \tau \) vs \( \delta \tau' \), \( \tau \) vs \( \Delta \tau + \delta \tau' \)

BEST TO MAKE \( n \) AS LARGE AS POSSIBLE, CONSISTENT WITH THE VARIOUS
CONSTRAINTS OF PULSE LENGTH, ETC., IF COMPUTER CAN HANDLE THE
INCREased PROCESSING

Statstics IRROVE EVEN THOUGH CLUTTER INCREASES —
ESPECIALLY IF S/N \( \ll 1 \) AND HENCE \( C \ll N \)

CANNOT USE ORTHOGONAL POLARIZATION TECHNIC TO ELIMINATE CLUTTER

CANNOT USE MULTIPLE FREQUENCIES EITHER (ELIMINATES CLUTTER, BUT
ALSO SIGNAL!)

SAME IDEA USED IN "NON-REDUNDANT" ANTENNA ARRAYS FOR APERTURE
SYNTHESIS. SOME REDUNDANCY OK IN ANTENNA ARRAY CASE, BUT NOT
FOR RADAR ACF MEASUREMENT (GIVES RANGE ALIASING)
INCOHERENT SCATTER

What is it?

Extremely weak scatter from electrons that are as unorganized as possible (usually not totally "incoherent" in some sense - but don't worry about this refinement now).

How weak?

10 km$^3$ with $10^{12}$ electrons/m$^3$

$\Rightarrow$ radar cross section $\sim 10^{-6}$ m$^2$

$\sim (1 \text{ mm})^2$

$(10^{-28}$ m$^2$ per electron)

But can be "easily" detected, nevertheless, with a powerful radar.

(W.E. Gordon, K.L. Bowles, 1958)

Theory

• Linear plasma kinetic theory
• Thoroughly worked out
• Very rich - spectrum of scattered signal depends on many plasma parameters

\[ \text{Power} \]

$\omega_0$  \hspace{2cm} $\omega$

Doppler shift $= \omega_p$

• Shape and total power $\Rightarrow N_e, T_e, T_i, V_d, \ldots \text{ etc.}$

ion composition (incl. negative ions)

$V_i$, differential ion drifts, currents ($V_e-V_i$)
How do we do the measurements?

- We want good resolution in range, time, and frequency.
- But target is "overspread", i.e., we want
  \[ \text{Doppler BW} \leq \text{IPP} \leq (2L/c) \text{, but BW} > (2L/c) \]
  (Nyquist) (Avoid echo overlap)
- A problem? Yes, but we can do clever things.

Various coding schemes and lag sequences used
- Individual pulses may be "compressed" (e.g. Barker coding)
IS (continued)

Choice of lag spacing or further coding

\[ \Rightarrow \]

statistical averaging

\[ \Downarrow \]

Range "clutter" (echoes from the "wrong" altitude) eliminated since echoes from different regions of space are uncorrelated

ERRORS

\[ \varepsilon^2 \sim \left( \frac{S+N+C}{S} \right)^2 \frac{1}{K} \]

\( K = \text{number of independent samples} \)
\( S = \text{signal power} \)
\( N = \text{noise power} \)
\( C = \text{clutter power} \)

ANALYSIS

Least square fitting of theory to data

\[ \Rightarrow \] \text{ionospheric parameters} \\
\text{Fit entire profile at one time? (OASIS program)} \]
IS (continued)

Questions:
- Non-Maxwellian plasmas (high latitudes)?
- IS from unstable plasmas (high latitudes)? Yes, apparently, for $k$ well inside stable regime.

What can we study using IS?
- Energy balance and $T_n$
- Photoelectron energy distribution, including arrivals from conjugate hemisphere
- Low and mid latitude winds, tides, gravity waves, TIDs
- $E$ fields, conductivities, dynamo theories
- High latitude winds, ion drag, magnetospheric forcing
- Magnetospheric convection, response to changes in the IMF and solar wind
- F region trough dynamics
- Ion chemistry, composition transitions
- Ionosphere-magnetosphere coupling (fluxes of particles and energy)
- Ion drift vs neutral winds (airglow)
- High latitude heating events (natural)
- Artificial (HF) heating experiments
Altitudes Covered and Resolution for IS

\( h: \sim 80 \text{ km} \) (even less sometimes)

\( \text{to several } \times 10^3 \text{ km} \)

\( \Delta h: \sim 150 \text{ m} \) (1 ms pulse) (Passion, coded pulses)

\( \text{to } \sim 150 \text{ km} \) (1 ms pulse) at high alts.

\( \Delta t: \sim \text{few sees to } \sim 1 \) hr
IS (continued)

- IS is the most powerful ground based technique for monitoring most of the important parameters of the ionosphere.

- Rapid improvements in DSP technology mean that we should soon be able to exploit the full potential of the method (years ago 90-99+ % of the data was sometimes wasted).

- There are many global IS observing programs associated with CEDAR, e.g. 
  GISMOS (substorms) 
  GITCAD (ionosphere-thermosphere coupling) 
  LTCS (lower thermosphere coupling) 
  SUNDIAL (not an acronym!) 
  WAGS (gravity waves) 
  CHARM (hydrogen) 
  MISETIA (equatorial) 
  ATLAS
RECENT TOPICS

- Non-Maxwellian ion distributions at high latitudes when drift velocity large.
- Can you observe IS looking along B while unstable waves are being generated with \( k \) nearly \( \perp B \)? Evidence seems to \( \Rightarrow \) Yes.
- Ionosphere is changing rapidly now with increasing solar activity.
- Digital data processing power improving rapidly. Cheap way to improve observatories.

NEW OBSERVATORIES?

- USSR EISCAT receiving station (VHF)
- Svalbard (Spitzbergen)? Japan? UK?
- Resolute Bay, Canada?
- Fairbanks, AK?
- Indonesia? (Japan)
- Other USSR observatories?

TIMETABLE?

- Any new high latitude radars should be ready by the time of the "Cluster" multisatellite launch in \( \sim 1995 \) if at all possible.
- So time is pretty short.