Numerical and analytical studies of critical radius in new geometries for corona discharge in air and CO$_2$-rich environments

Jacob A. Engle, Jeremy A. Riosset

Department of Physical Sciences, Center for Space and Atmospheric Research (CSAR), Embry-Riddle Aeronautical University, Daytona Beach, FL

engle2@myeru.edu

Abstract

In this work, we focus on plasma discharge produced between two electrodes with a high potential difference, resulting in ionization of the neutral gas particles and creating a current in the gas. This process, when done at low current and low temperature, can create corona and glow discharges, which can be observed as a luminous, or "glow" discharge. The parallel plate geometry used in Paschen theory is particularly well suited to model experimental laboratory scenario. However, it is limited in its applicability to lightning and power lines (Moore et al., 2000). Franklin's sharp tips and Moore et al.'s rounded tips fundamentally differ in the radius of curvature of the upper and bottom end of the rod. Hence, we propose to expand the classic Cartesian geometry into spherical geometries. In spherical case, a small radius effectively represents a sharp tip rod, while larger, centimeter-scale radius represents a rounded, or blunter tip. Experimental investigations of lightning-like discharge are limited in size. They are typically either a few meters in height, or span along the ground to allow the discharge to develop over larger distance. Yet, neither scenario account for the change in pressure, which conditions the reduced electric field, and therefore hardly reproduce the condition of discharge. A useful approximation used in research is the Paschen curve, which describes the relationship between the electric field and the gap distance for a given gas pressure. We also explore the effects of the shifting from the classic parallel plate analysis to spherical and cylindrical geometries more adapted for studies of lightning rods and power transmission lines, respectively. Utilizing Townsend equation for corona discharge, we estimate a critical radius and minimum breakdown voltage that allows ionization of neutral gas and formation of a glow corona around an electrode in air. Additionally, we explore the influence of the gap in which the discharge develops. We use Bolzmann, a numerical solver for the Boltzmann equation, to calculate Townsend coefficients for CO$_2$-rich atmospheric conditions (Hagelar and Pitchford, 2005). This allows us to explore the feasibility of a glow corona on other planetary bodies such as Mars. We calculate the breakdown criterion both numerically and analytically to present simplified formulas and approximate tandem leaders unique in electric discharge.

I. Introduction

Corona Discharge

- Electrical discharge around a conductor due to electric field;
- Weakly ionized gas responsible for glow at visible wavelengths;
- Hypothesized to promote the formation of upward-branching leaders in lightning discharges.

Electron Avalanche

- The process of electron avalanche is similar between various types of discharges:
  - Initial step of a discharge;
  - Release of secondary electrons in electron-neutral collision;
  - Secondary electrons with enough KE to repeat the process;

Avalanche criteria: [Raizer, 1991]

\( \int_0^\infty \alpha d\nu = \ln(\nu_0) = 10^{-20}; \nu = 10^{8} \)

Types of Discharges

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Glow Corona</th>
<th>Streamer</th>
<th>Leader</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>~300 K</td>
<td>~300 K</td>
<td>≥5000 K</td>
</tr>
<tr>
<td>Electron energy</td>
<td>1-2 eV</td>
<td>5-15 eV</td>
<td>1-2 eV</td>
</tr>
<tr>
<td>Effective electric field</td>
<td>0.2-2.7 kV/cm</td>
<td>5-7.5 kV/cm</td>
<td>1.5 kV/cm</td>
</tr>
<tr>
<td>Electron density</td>
<td>2-6·10$^{12}$ cm$^{-3}$</td>
<td>5·10$^{13}$-10$^{14}$ cm$^{-3}$</td>
<td>4·10$^{15}$ cm$^{-3}$</td>
</tr>
</tbody>
</table>

Table 1: Classification of types of discharge at low (Earth) and high (Mars) pressures.

II. Model Formulation

Objective

- Apply Paschen theory to Cartesian, spherical, and cylindrical geometries;
- Obtain analytical expressions for critical radius and Stoletov's point;
- Develop numerical models for Cartesian, spherical, and cylindrical geometries;
- Verify numerical models and analytical solutions with experimental data;
- Establish the differences between sharp- and blunt-tipped rods for corona discharges;
- Generalize to any atmosphere using a Boltzmann solver (Hagelar and Pitchford, 2005).

III. Results

Cartesian solutions

- Critical electric field: \( E_d = \frac{Bp}{\ln(\nu_0) \cdot \nu} \), where \( E_d > 0 \);
- Minimum breakdown voltage:
  \( V_d = \frac{4Bp}{1-\nu} \ln(\nu_0) \cdot \nu \);
- Stoletov's point: \( V_{min} = \frac{Bp}{\nu} \ln(\nu_0) \);
- Largest error due to Taylor expansion of Gauss error function;
- Boltzmann equation solver (Bolz)
- Highest minimum breakdown voltage

Spherical solutions

- Critical electric field: \( E_s = \frac{Bp}{\ln(\nu_0) \cdot \nu} \), where \( E_s > 0 \);
- Minimum breakdown voltage:
  \( V_s = \frac{4Bp}{1-\nu} \ln(\nu_0) \cdot \nu \);
- Stoletov's point: \( V_{min} = \frac{Bp}{\nu} \ln(\nu_0) \);
- Solution not valid for large radius
- Boltzmann equation solver (Bolz)

Cylindrical solutions

- Critical electric field: \( E_c = \frac{Bp}{\ln(\nu_0) \cdot \nu} \), where \( E_c > 0 \);
- Minimum breakdown voltage:
  \( V_c = \frac{4Bp}{1-\nu} \ln(\nu_0) \cdot \nu \);
- Simplification using the LambertW function
- Solutions not valid for large radius
- Boltzmann equation solver (Bolz)

IV. Conclusions

The results and conclusions obtained in this work can be summarized as follows:

- A new model for calculations of the critical radius and minimum breakdown voltage for corona discharge in Cartesian, spherical, and cylindrical geometries is validated.
- The model is validated using classical Paschen theory and experimental data in air and CO$_2$ and CO$_2$-rich discharges.
- We expand classical Paschen theory into an analytical solution for spherical and cylindrical geometries.
- Our numerical model and the analytical solution show excellent agreement with experimental data.
- The significantly lower pressure on Mars compared to Earth lowers the minimum breakdown voltage required to create corona discharge.

Acknowledgements

This work is supported by the Embry-Riddle Aeronautical University Office of Undergraduate Research (EARU) and the Center for Space and Atmospheric Research (CSAR).

REFERENCES


Table 2: Exponential approximation coefficients (A and B) from Figure 5 found from fitting \( \alpha_{eff} = A \exp(-B \nu) \).