Modeling of Gravity Wave and Instability Processes
in the Middle Atmosphere

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Outline

1. Motivations for MA Instability Studies
2. Model Formulation
3. Instability due to Wave Breaking
4. Kelvin-Helmholtz Instability
5. Conclusions
Modeling Motivations

- Wave transports of energy and momentum are central to our understanding of middle atmosphere dynamics

- Wave interaction and instability processes account for wave saturation, spectral character, and constraints on energy and momentum fluxes

- Dynamics of transition from laminar to turbulent flow dictates character of turbulence, efficiency of mixing and transports
**Model Formulation**

- Solves Euler equations with spectral viscosity

- Employs spectral collocation techniques
  - Fourier in $x$, $y$
  - Chebyshev in $z$

- Uses domain decomposition for higher resolution, greater efficiency
  - wave breaking using two domains
    - forcing in low-resolution lower domain
      (96, 48, 65)
    - instability in high-resol. upper domain
      (192, 96, 129)
  - Kelvin-Helmholtz instability using four domains
    - Re = 200 to 2000

- 2D initial evolution, 3D instability evolution following noise insertion at finite amplitude

- Boundary and interface conditions
  - periodic in $x$, $y$
  - open in $z$, using upstream characteristics
Wave Breaking Simulations

- high-frequency wave in a shear flow
  - ~ 30 min period
  - ~ 24 km wavelength
  - ~ 1 km instability depth

- wave field evolution
  - initial instability is convective, streamwise
  - secondary instability is dynamical, spanwise and localized (3D KH)
  - evolution is rapid and transient, collapse to turbulence ~ 1 T_b
Wave breaking shown with isosurface of 0
wave breaking with isosurface of $O$
and of positive (red) and negative (blue)
streamwise vorticity
Eddy Kinetic Energy Equation

\[
\left( \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) K_e + \frac{\partial}{\partial x} \langle p'u' \rangle + \frac{\partial}{\partial z} \langle p'w' \rangle
\]

\[
\approx -\hat{\rho} \langle u'u'_i \rangle \frac{\partial}{\partial x} \tilde{u}_i - \hat{\rho} \langle u'_i w' \rangle \frac{\partial}{\partial z} \hat{u}_i + \frac{\hat{\rho} g}{\theta} \langle \theta'w' \rangle
\]

Vorticity Equation

\[
\frac{d\omega_i}{dt} \approx \omega_j S_{ij} + \left\{ \frac{\nabla \rho}{\rho} \times \frac{\nabla p}{\rho} \right\}_i
\]

where

\[
S_{ij} = \frac{1}{2} (\partial_i v_j + \partial_j v_i)
\]
eddy momentum fluxes (domain averaged)

\[ \overline{\nu'w'} \]  

eddy heat fluxes (domain averaged)

\[ \overline{0'w'} \]
Modeling of Breaking Gravity Wave

- Vortices rendered by $\lambda_2 < 0$ of $S^2 + R^2$, viewed from below
Baroclinic generation of vortices at $t=62.5$
Vortices at $t=67.5$, strain source of streamwise vorticity $(\omega_j S_{ij})_1$
**Kelvin-Helmholtz Instability**

- unstable shear flow in uniform stratification
  - \( U(z) = U_0 \tanh(z/h) \), \( U_0 = 28 \text{ m/s} \), \( h = 300 \text{ m} \)
  - wavelength \(~ 4 \text{ km}\)
  - \( Ri = \frac{N^2}{Uz^2} = 0.05 \)
  - \( Re = 200 \text{ to } 2000 \)

- KH evolutions
  - remain 2D, \( Re < 200 \)
  - secondary convective instability, \( Re > 250 \)
  - secondary dynamical instability, \( Re > 1000 \)
  - secondary instabilities
    - accelerate KH breakdown, restratification
    - mixing and transports are very different in 2D and 3D
Contour of $\theta = 1.035$ for $Re = 500$
Contours of positive (red) and negative streamwise vorticity for Reynolds number = 500
Re = 500 Potential Temperature

2D
Time = 8

3D
Time = 8

Time = 16

Time = 16

Time = 24

Time = 24

Time = 32

Time = 32

Time = 40

Time = 40

0. .72

0. .72
Re = 500 Spanwise Vorticity

2D
Time = 8

Time = 16

Time = 24

Time = 32

Time = 40

0.  .72

3D
Time = 8

Time = 16

Time = 24

Time = 32

Time = 40

0.  .72
Re = 500 Spanwise Vorticity

2D
Time = 48

Time = 56

Time = 64

Time = 72

Time = 80

3D
Time = 48

Time = 56

Time = 64

Time = 72
KH Mean Flow Evolutions

\[ \langle uz \rangle \text{ Profile } \text{Re} = 500 \]

\[ \overline{u(z)} \text{ Mean velocity profile } \text{Re} = 500 \]

\[ \langle \theta z \rangle \text{ Profiles for } \text{Re} = 500 \]

\[ \overline{\theta(z)} \text{ Re = 500} \]
Conclusions

- Wave breaking is inherently three dimensional
  - primary instability is convective in nature over large range of wave frequencies
  - secondary dynamical instability (KH in 3D) arises due to stretching of vortex sheets
  - vorticity dynamics drives transition to turbulence
    - intertwined vortex tubes
    - intense vortex interactions
    - vortex fraying, fragmentation => cascade of energy and enstrophy to smaller scales

- Kelvin-Helmholtz instability exhibits secondary instability
  - convective, streamwise instability, Re > 250
  - dynamical, spanwise aligned inst., Re > 1000
  - 2D and 3D evolutions have very different
    - vorticity dynamics
    - implications for mixing and transports