DATA ASSIMILATION OF RADIATION BELT ELECTRONS USING MULTI-POINT OBSERVATIONS AND THE VERB CODE: 
LESSONS LEARNED AND FUTURE DEVELOPMENTS 

ADAM C KELLERMAN, Y. Y. SHPRITS, D. A. KONDRASHOV,
T. PODLADCHIKOVA, A. Y. DROZDOV, H. ZHU
Kalman Filter and Errors

X: State vector, PSD (c/(cm.MeV))³  
M: Model matrix (VERB code)  
P: State error covariance matrix  
Q: Model covariance matrix  
y: PSD measurements  
K: Kalman Gain  
R: Measurement error

**Forecast Step:**

\[
X_f = M_t X_{t-1}
\]

\[
P_f = M_t P_{t-1} M_t^T + Q_t
\]

**Update Step**

\[
X_a = X_f + K_t (y_t - X_f)
\]

\[
K_t = P_f (P_f + R_t)^{-1}
\]

\[
P_a = (I - K_t) P_f
\]
Operator Splitting in 2D

\[ X_t^f = M_{t-1} X_{t-1}^f \]

\[ X_t^f = M_{t-1} \alpha M_{t-1} \varepsilon X_{t-1}^f \]

“Standard” Kalman Filter

“Operator-Splitting” Kalman Filter
In this study, we use the Versatile Electron Radiation Belt (VERB) code Diffusion code that solves the Fokker-Planck equation for electron phase space density (PSD)
Some important lessons learned and future developments

1. Operator splitting may be used to approximate a full 3D Kalman filter approach.
2. Statistical errors do not result in significant differences in model forecast performance – in future we require an activity dependent approach, and perhaps more attention to spatial dependence.
3. With the current framework we can not determine innovation, and interpretation of the 3 sets of covariance matrices is difficult – we are limited to improving the model through statistical error analysis and physical understanding.
4. The method is however suitable for fast operation and performs quite well in an operational framework.

Future:
1. Introduce 2D Kalman filtering in pitch angle and energy – this should allow us to improve our wave-particle interaction models.
2. Implement more sophisticated ways of accounting for errors and test.
Error Analysis – Models and Data

We have computed errors in electron PSD using several B-field models and a PSD matching technique.
Error Analysis – Models and Data

Old Diffusion Coefficients
See Subbotin et al., [2010] JGR

New Diffusion Coefficients
Hiss - Spasojevic et al, [2015] and Orlova et al, [2016]
Validation metrics

\[ PE = 1 - \frac{\sum_{i=1}^{N} (m_i - p_i)^2}{\sum_{i=1}^{N} (m_i - \langle m_i \rangle)^2}. \]

\[ SS = \frac{PE_{Model} - PE_{Persist}}{1 - PE_{Persist}} = \frac{\sum_{i=1}^{N} (m_i - m_{i-1})^2 - \sum_{i=1}^{N} (m_i - p_i)^2}{\sum_{i=1}^{N} (m_i - m_{i-1})^2}. \]

\[ FS = \frac{PE_{Model}}{PE_{Persist}} = \frac{\sum_{i=1}^{N} (m_i - \langle m_i \rangle)^2 - \sum_{i=1}^{N} (m_i - p_i)^2}{\sum_{i=1}^{N} (m_i - \langle m_i \rangle)^2 - \sum_{i=1}^{N} (m_i - m_{i-1})^2}. \]
Forecast Performance

Old Diffusion Coefficients

PSD in a 15 min interval, one day ahead

New Diffusion Coefficients

PSD in a 15 min interval, two days ahead
Performance Analysis – one day

Equal errors

Statistical errors
Performance Analysis – one day

Old Dxx – Equal errors

New Dxx – Equal errors
Performance Analysis – one day

New Dxx – Equal errors

New Dxx – statistical errors
Performance Analysis – two day

New Dxx – Equal errors

New Dxx – statistical errors