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Tutorial Lecture

by Jeffrey Forbes
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Tidal and Planetary Waves

TIDAL AND PLANETARY WAVES

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- **Motivation: MLT Observations**

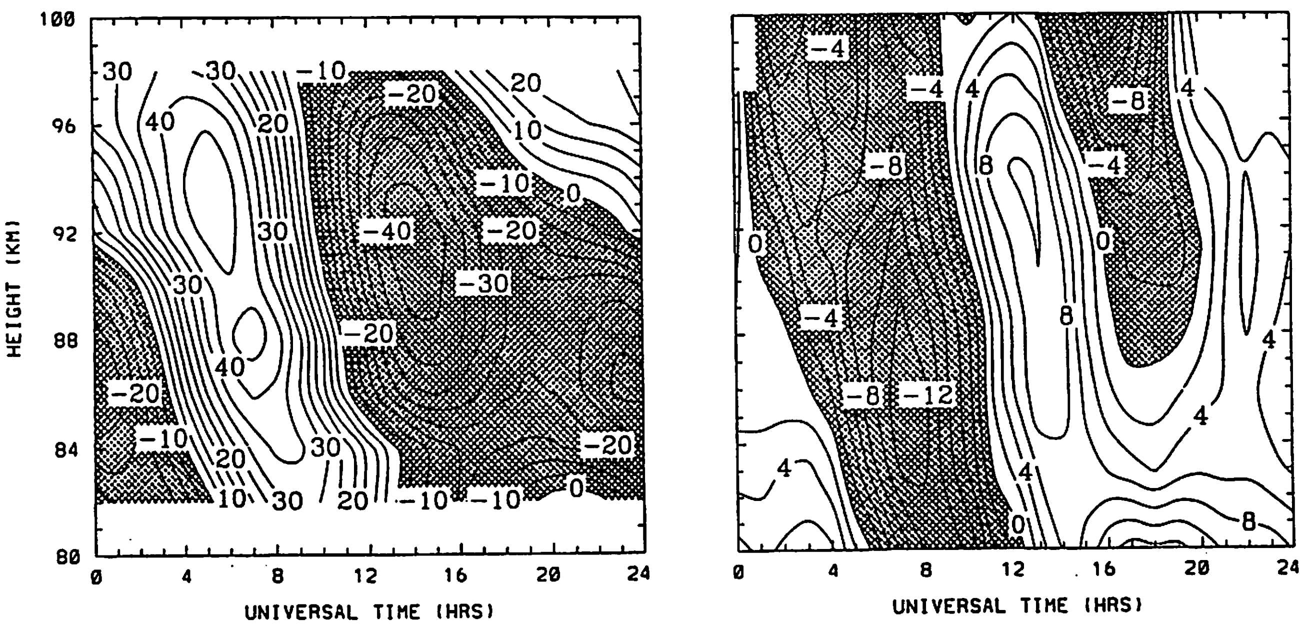
- **Theory of atmospheric "forced" and "free" oscillations (analytic; isothermal atmosphere)**

- **Effects of non-isothermality, dissipation, and mean winds; MLT modeling efforts**

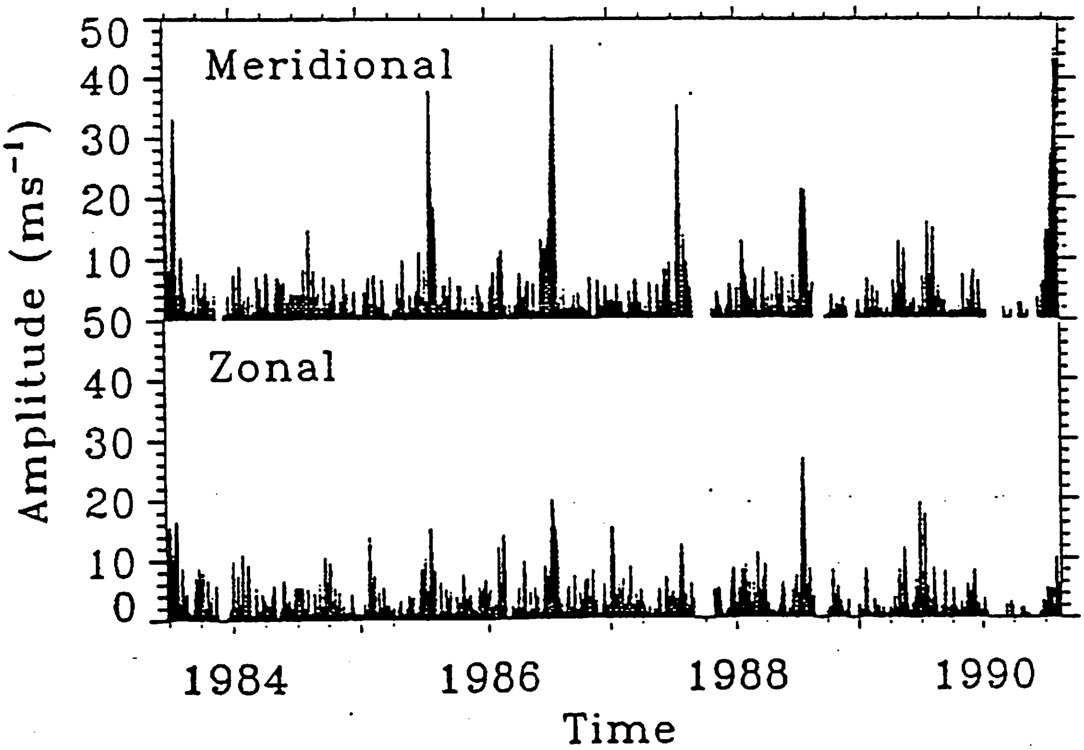
- **Future possibilities for MLT studies**

Average Meridional Winds, March 18-27, 1979

Townsville (19°S, 147°E) Saskatoon (54°N, 107°W)



2-Day Wave over Adelaide
Harris [1993]



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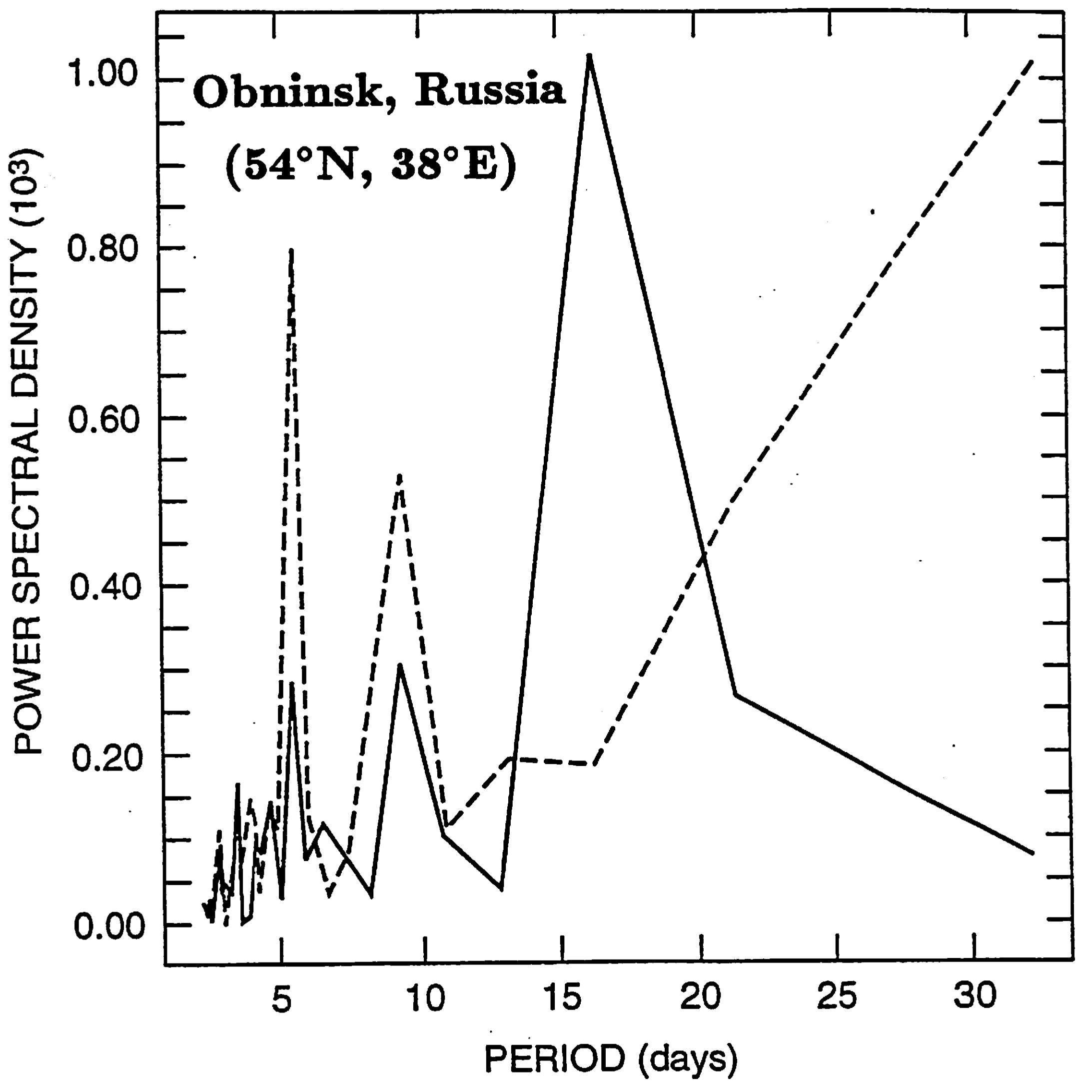
Spectrum of Daily Mean Winds

January-February, 1979



In the absence of mean winds, the linearized equations for atmospheric perturbations on a sphere are [Holton, 1975]:

$$\frac{\partial u}{\partial t} - 2\Omega \sin\theta v + \frac{1}{a\cos\theta}\frac{\partial \Phi}{\partial \lambda} = 0 \tag{1}$$

$$\frac{\partial v}{\partial t} + 2\Omega \sin\theta u + \frac{1}{a}\frac{\partial \Phi}{\partial \theta} = 0 \tag{2}$$

$$\frac{\partial}{\partial t}\Phi_z + N^2 w = \frac{\kappa J}{H} \tag{3}$$

$$\frac{1}{a\cos\theta}\left[\frac{\partial u}{\partial \lambda} + \frac{\partial}{\partial \theta}(v\cos\theta)\right] + \frac{1}{\rho_o}\frac{\partial}{\partial z}(\rho_o w) = 0 \tag{4}$$

u = eastward velocity
v = northward velocity
Φ = geopotential
$w = dz/dt$
N^2 = buoyancy frequency squared
Ω = angular velocity of earth
ρ_o = basic state density
$z = -H ln(p/p_o)$
λ = longitude
θ = latitude
t = time
$\kappa = R/c_p \approx \frac{2}{7}$
J = heating per unit mass
a = radius of earth
H = constant scale height

Now assume the perturbations to consist of longitudinally propagating waves of zonal wavenumber s and frequency σ:

$$\{u, v, \Phi\} = \{\hat{u}, \hat{v}, \hat{\Phi}\} exp[i(s\lambda - \sigma t)]$$

Consolidation results in a single $z - \theta$ equation with separable solutions:

$$\hat{\Phi} = \sum_n \Theta_n(\theta) G_n(z)$$

$$\hat{J} = \sum_n \Theta_n(\theta) J_n(z)$$

$$\hat{u} = \frac{\sigma}{4\Omega^2 a} \sum_n U_n(\theta) G_n(z)$$

$$\hat{v} = \frac{-i\sigma}{4\Omega^2 a} \sum_n V_n(\theta) G_n(z)$$

Where $\{\Theta_n\}_{all\ n}$ is a complete orthogonal set.

Expressions for U_n and V_n are derived from the momentum equations:

$$U_n = \frac{1}{(f^2 - \sin^2\theta)}[\frac{s}{\cos\theta} + \frac{\sin\theta}{f}\frac{d}{d\theta}]\Theta_n$$

$$V_n = \frac{1}{(f^2 - \sin^2\theta)}[\frac{s\tan\theta}{f} + \frac{d}{d\theta}]\Theta_n$$

The condition for separability is as follows:

$$i\sigma[\frac{1}{\rho_o}\frac{\partial}{\partial z}(\frac{\rho_o}{N^2})\frac{\partial}{\partial z}G_n] + \frac{1}{\rho_o}\frac{\partial}{\partial z}(\frac{\rho_o \kappa J_n}{HN^2}) = -\frac{i\sigma}{gh_n}G_n$$

where h_n is the so-called equivalent depth.

Defining $G'_n = G_n \rho^{1/2} N^{-1}$, and assuming an isothermal atmosphere for which $N^2 = \frac{\kappa g}{H}$ where H = constant $\approx$ 7.5 km, and letting $x = z/H$, results in the so-called *vertical structure equation* (for an isothermal atmosphere):

$$\frac{d^2 G'_n}{dx^2} + [\frac{\kappa H}{h_n} - \frac{1}{4}]G'_n = -\frac{\rho_o^{-1/2}}{i\sigma N}\frac{d}{dx}(\rho_o \kappa J_n)$$

The θ-dependent part of the solution is embodied in the so-called *Laplace's tidal equation*:

$$\frac{d}{d\mu}[\frac{(1-\mu^2)}{(f^2-\mu^2)}\frac{d\Theta_n}{d\mu}] - \frac{1}{f^2-\mu^2}[-\frac{s}{f}\frac{(f^2+\mu^2)}{(f^2-\mu^2)} + \frac{s^2}{1-\mu^2}]\Theta_n + \epsilon\Theta_n = 0$$

where $\mu = \sin\theta$ and $\epsilon_n = (2\Omega a)^2/gh_n$.

Vertical Structure Equation

$$\frac{d^2G'_n}{dx^2} + \alpha^2 G'_n = F(x)$$

$$\alpha^2 = \frac{\kappa H}{h_n} - \frac{1}{4}$$

$$G'_n \sim Ae^{i\alpha x} + Be^{-i\alpha x}$$

I. 'FORCED' SOLUTION

(a) "Evanescent" or "Trapped" Wave: If $h_n < 0$ or $h_n > 4\kappa H$, then $\alpha^2 < 0$ and for a bounded solution above the source region:

$$G'_n \sim e^{-|\alpha|x}$$

(b) "Propagating" Wave: If $0 < h_n < 4\kappa H$, then $\alpha^2 > 0$ and "radiation condition" ($C_{gx} > 0$) at $x = \infty$ implies

$$G'_n \sim e^{\pm i|\alpha|x}$$

where (+,-) corresponds to (westward,eastward) propagating waves.

II. 'FREE' SOLUTION (F(x) = 0)

Only nontrivial solution satisfying boundedness and w = 0 at z = 0:

$$G'_n \sim e^{(\kappa - \frac{1}{2})x}$$

and

$$h_n = \frac{H}{1-\kappa}$$

where h_n = 10.5 km for H = 7.5 km. This free (unforced) solution corresponds to a resonant response of the atmosphere. Note that the above solution implies

$$u \sim e^{\kappa x}$$

corresponding to energy decay away from the surface (ρu^2) while velocity and other wave fields increase exponentially (by a factor of 40 from the surface to 100 km). These waves are sometimes called 'Lamb' or 'edge' waves.

Laplace's Tidal Equation

$$F_{s,\sigma}(\Theta_n^{s,\sigma}) = \epsilon_n^{s,\sigma}\Theta_n^{s,\sigma}$$

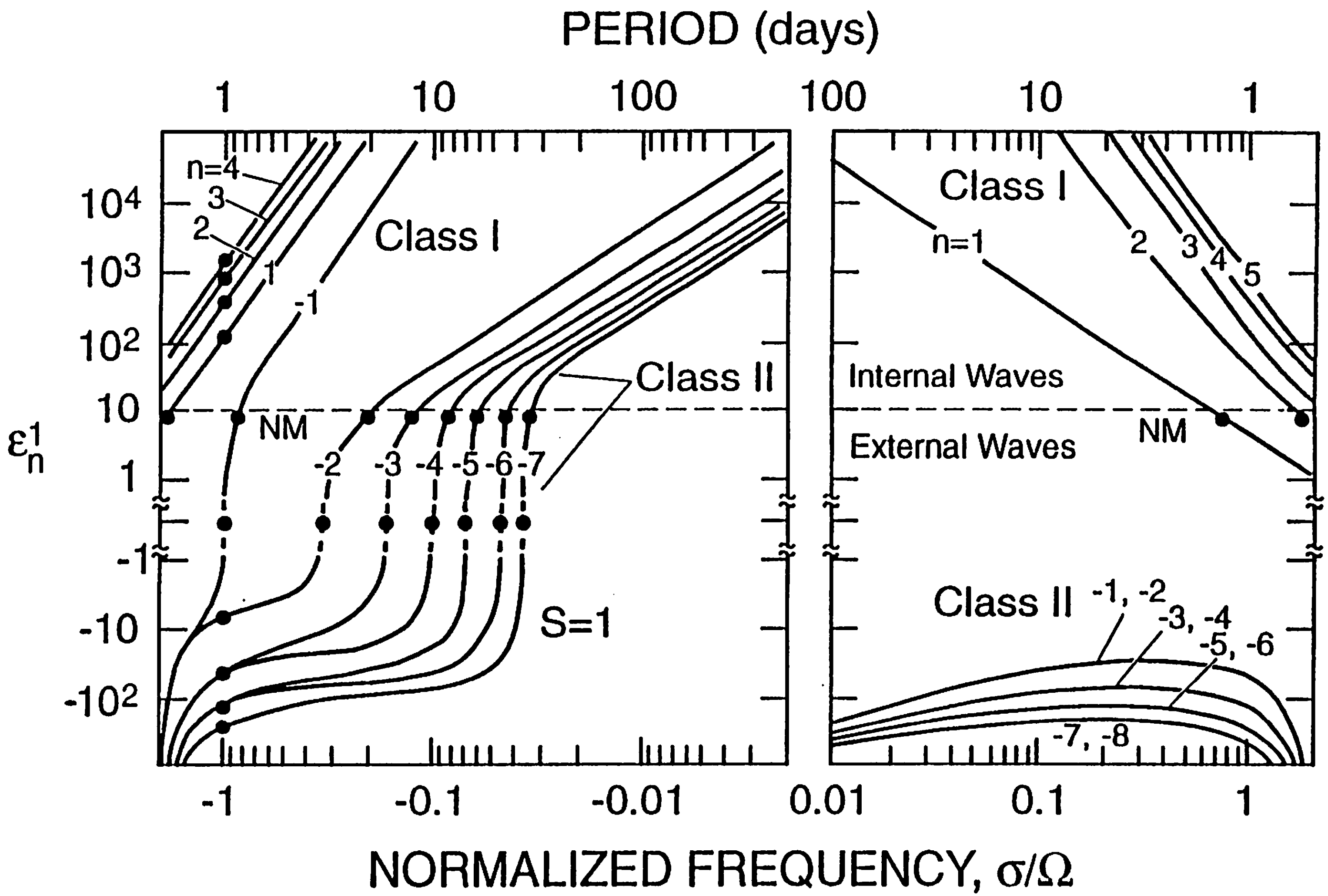
For each choice of s and σ, there exists a series of ϵ_n and Θ_n.

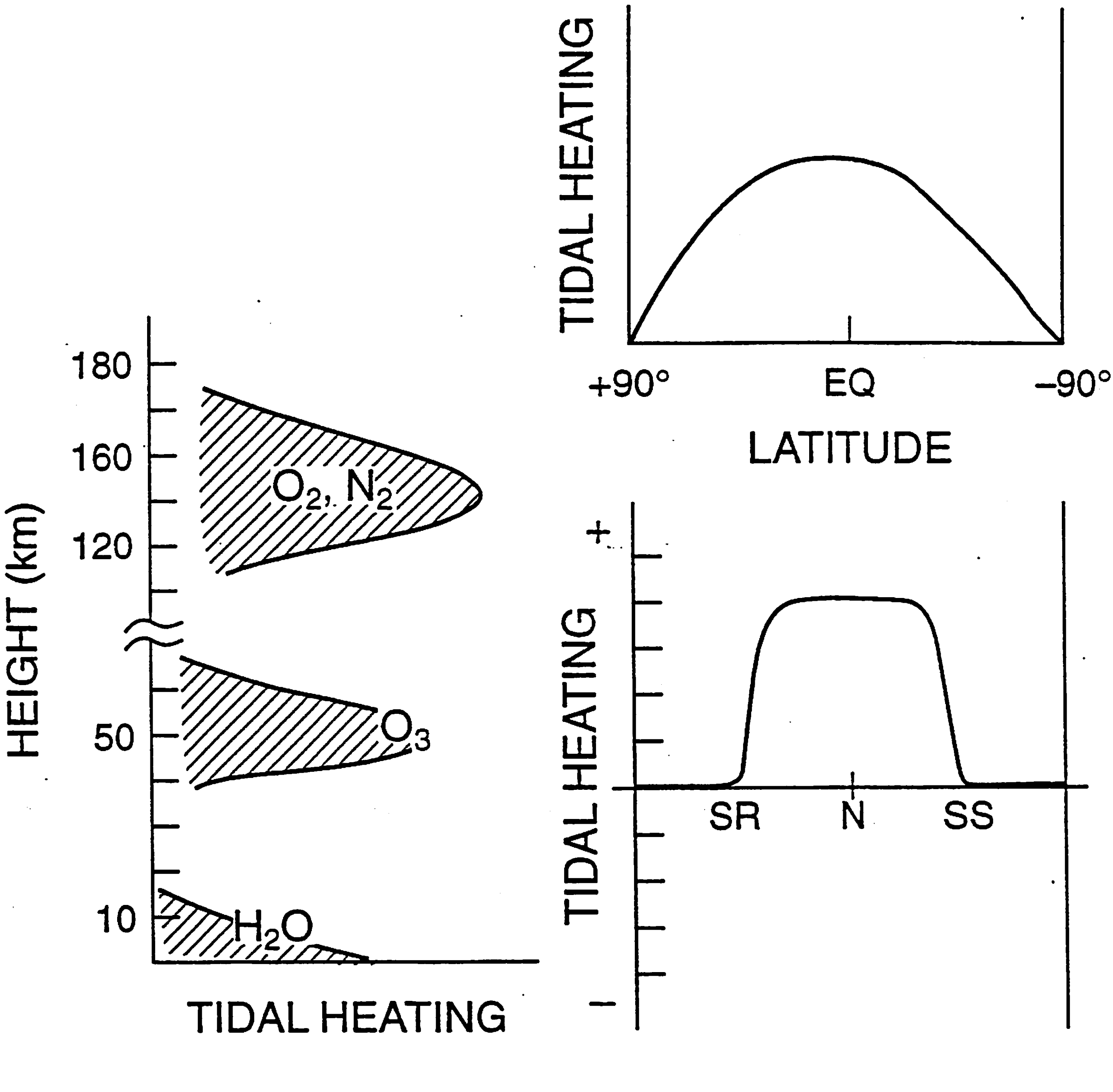
"Solutions of First Kind"	**"Solutions of Second Kind"**
("Class I")	**("Class II")**
("Gravity Modes")	**("Rotational or Rossby Modes")**
$\epsilon > 0$ **for** $\frac{\sigma}{s} > 0$	$\epsilon > 0$ **or** $\epsilon < 0$ **for** $\frac{\sigma}{s} < 0$
$\epsilon > 0$ **for** $\frac{\sigma}{s} < 0$	$\epsilon < 0$ **for** $\frac{\sigma}{s} > 0$

$\frac{\sigma}{s} > 0$ **for eastward propagating wave**

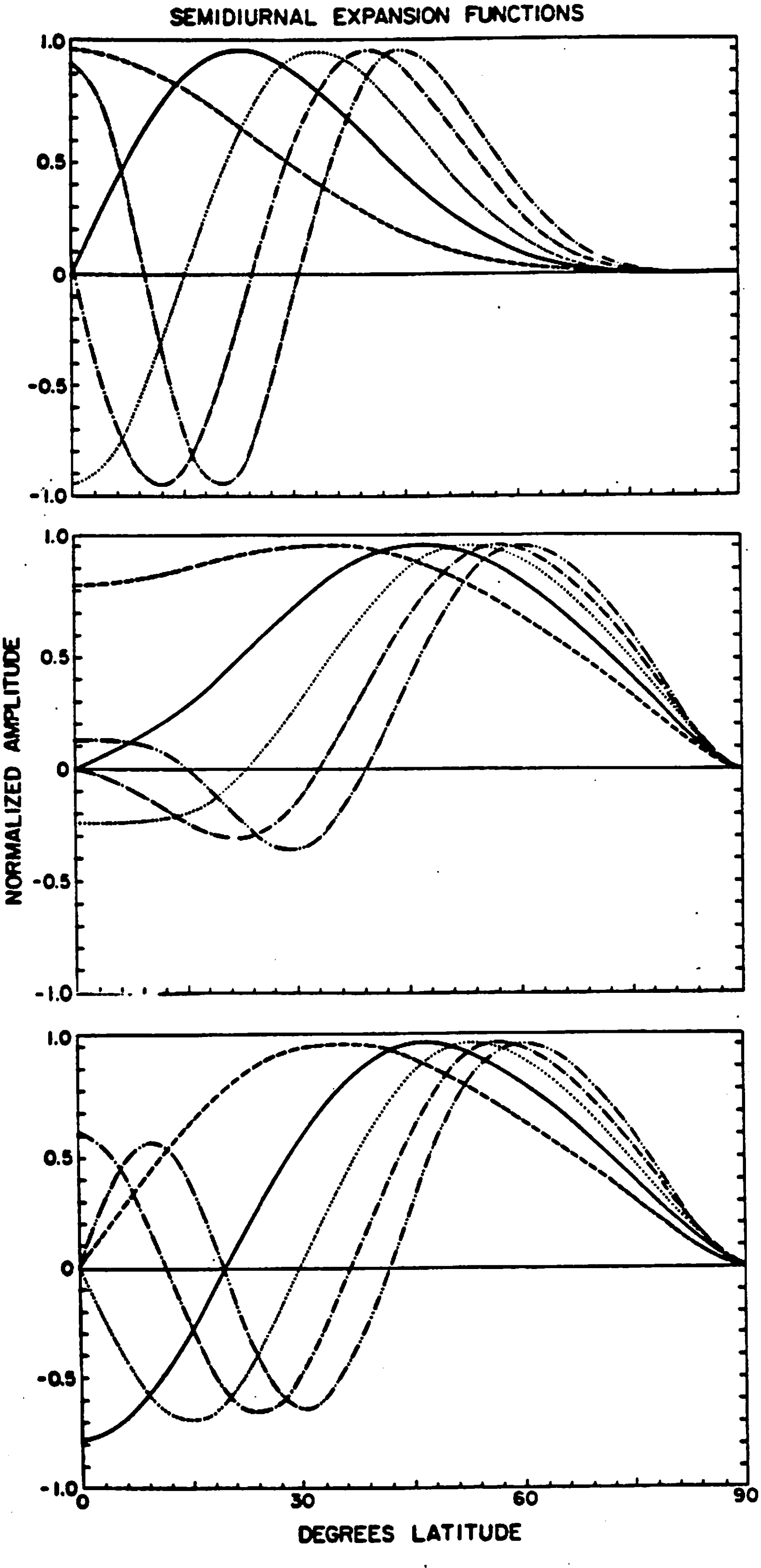
$\frac{\sigma}{s} < 0$ **for westward propagating wave**

Volland [1988]

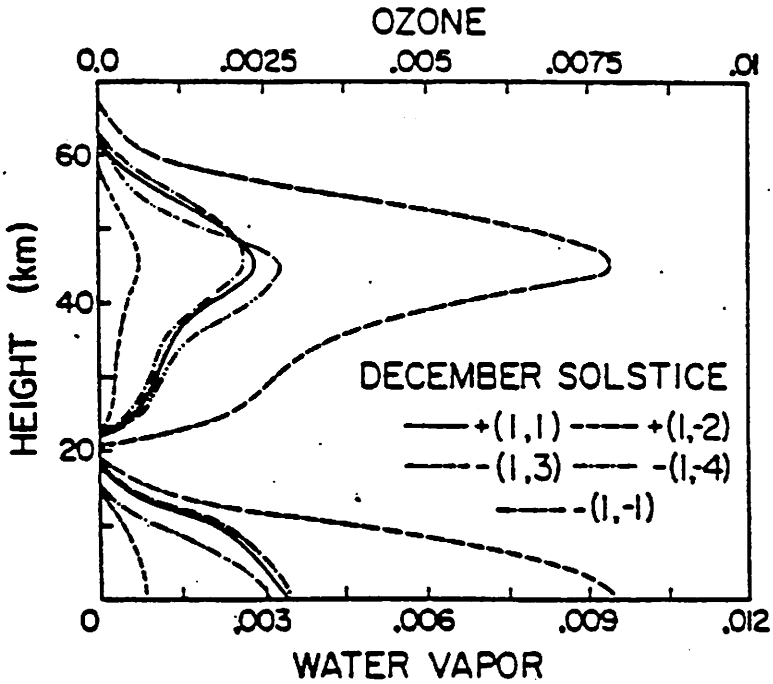


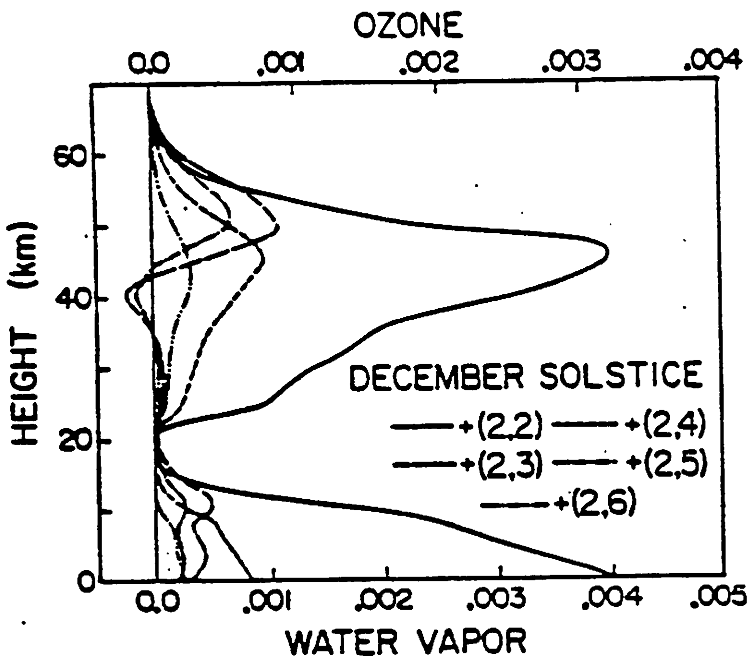


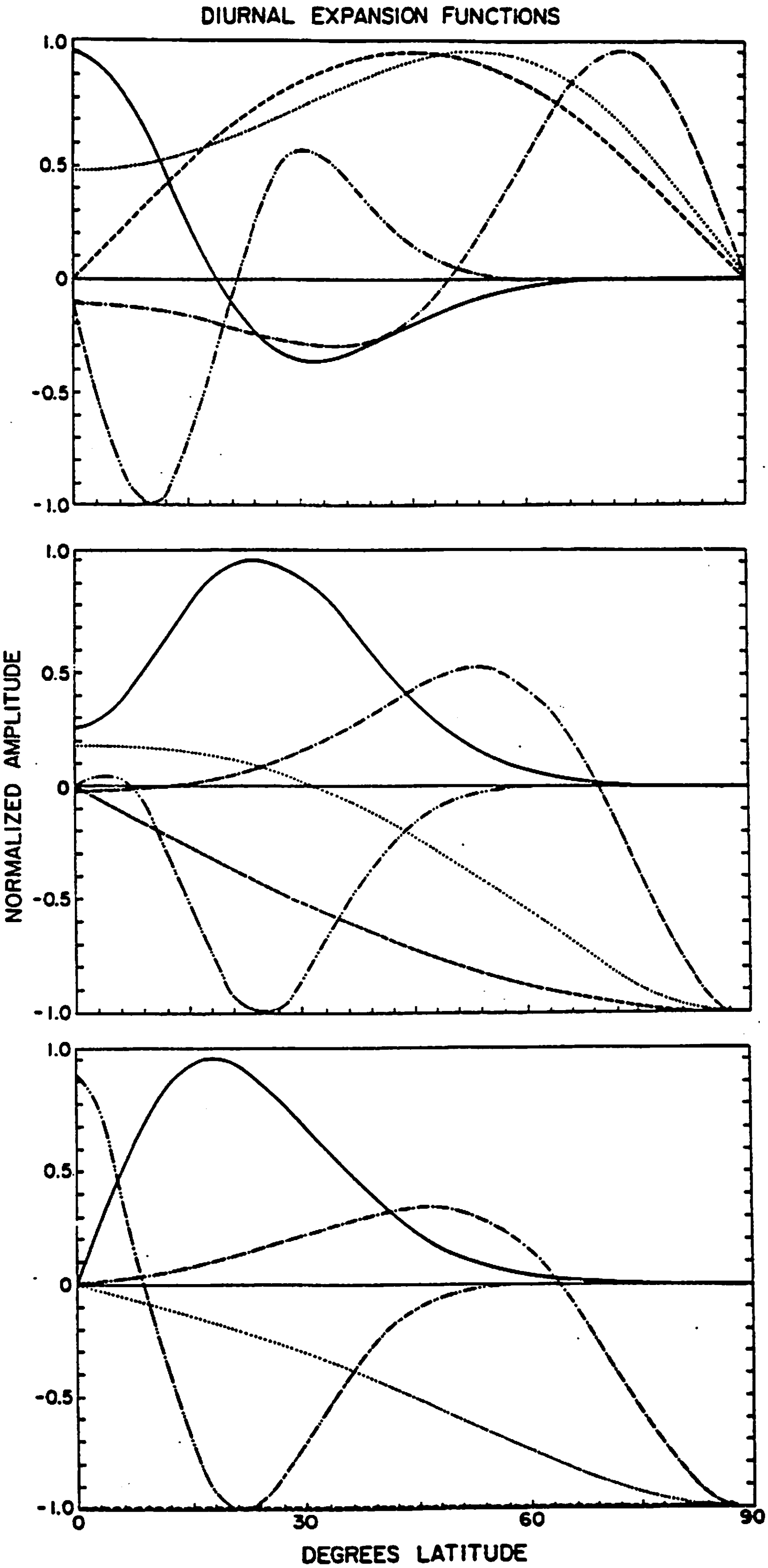
FORCING OF ATMOSPHERIC TIDES



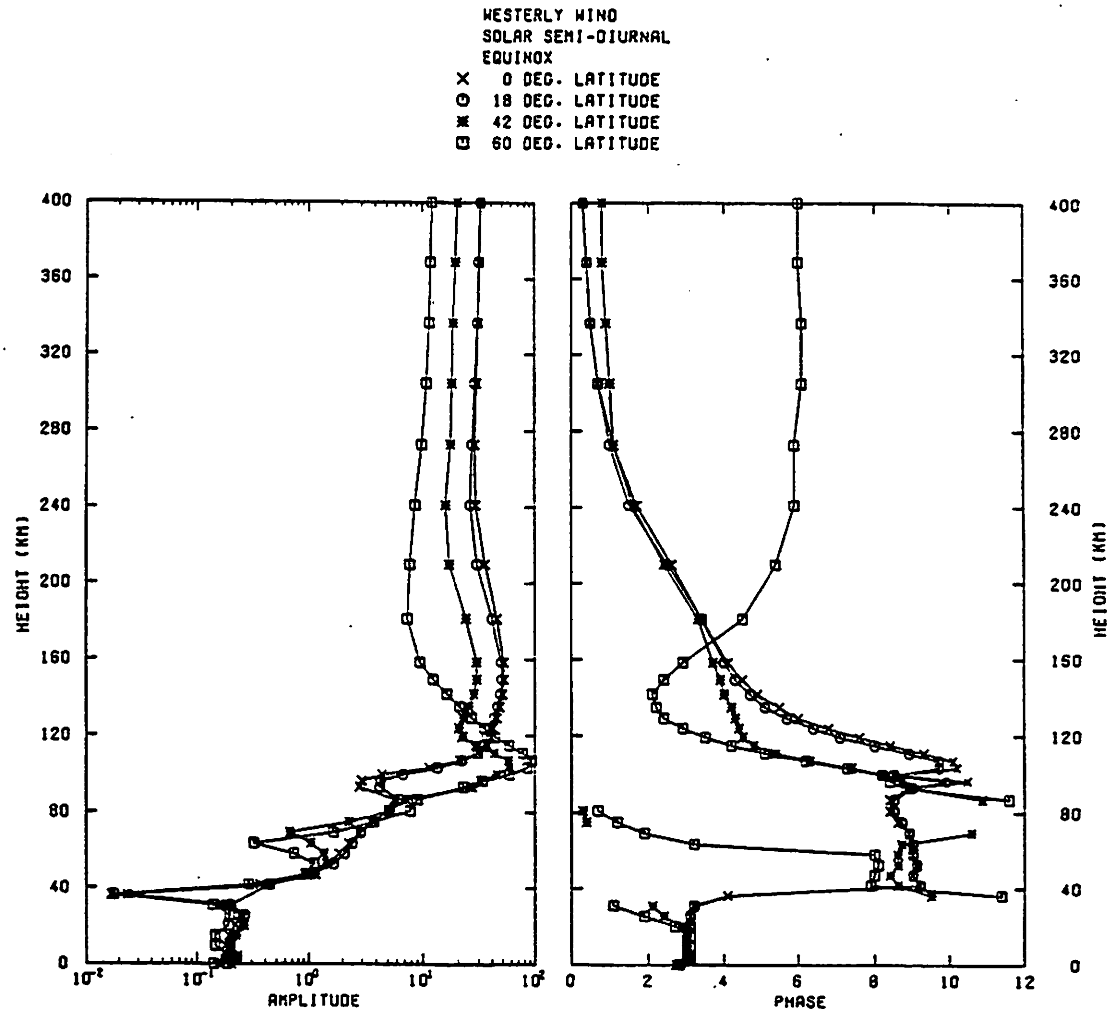
SEMIDIURNAL EXPANSION FUNCTIONS
NORMALIZED AMPLITUDE
1.0
0.5
0
-0.5
-1.0
0
30
60
90
DEGREES LATITUDE

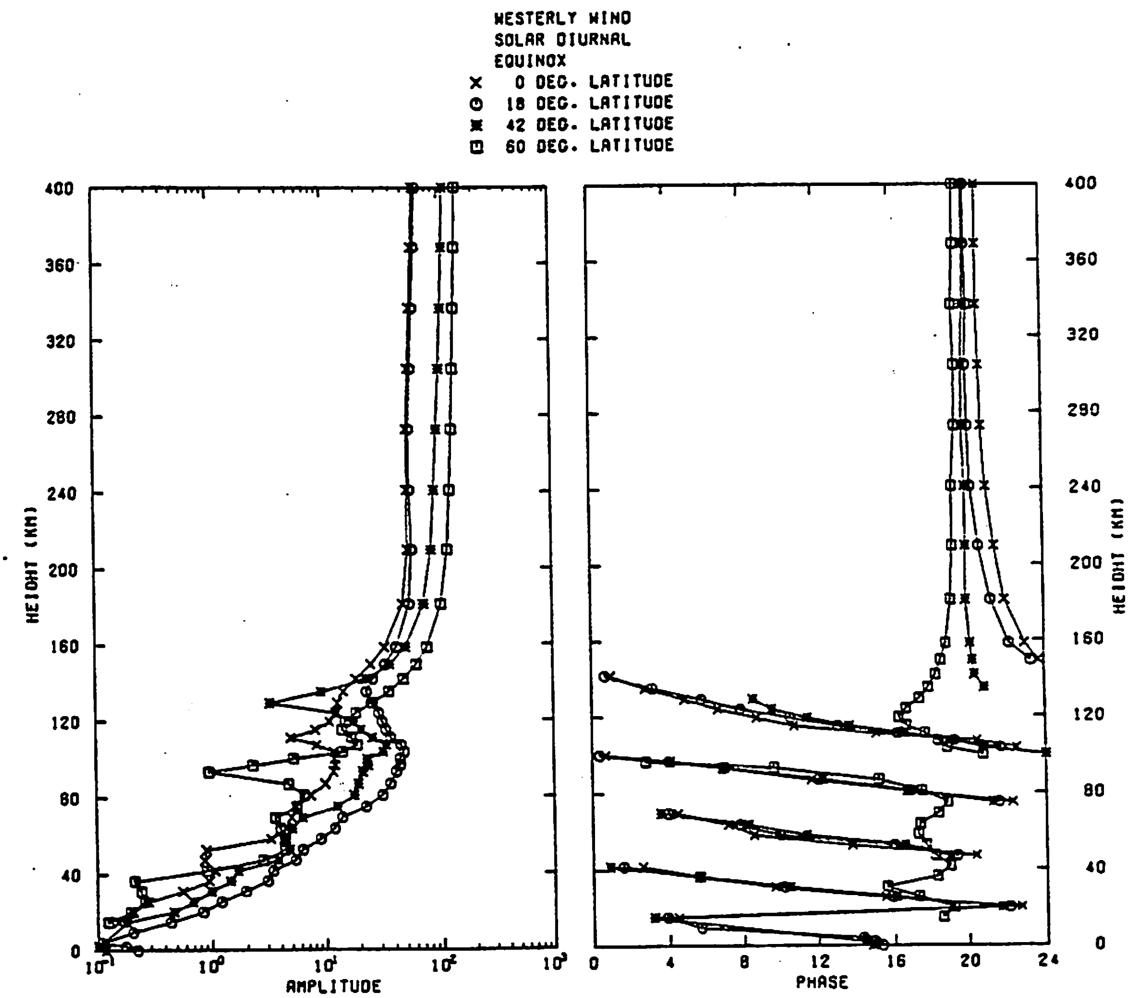

OZONE
0.0
.0025
.005
.0075
.01
HEIGHT (km)
60
40
20
0
.003
.006
.009
.012
WATER VAPOR
DECEMBER SOLSTICE
+(1,1)
+(1,-2)
-(1,3)
-(1,-4)
-(1,-1)


OZONE
0.0
.001
.002
.003
.004
HEIGHT (km)
60
40
20
0
0.0
.001
.002
.003
.004
.005
WATER VAPOR
DECEMBER SOLSTICE
+(2,2)
+(2,4)
+(2,3)
+(2,5)
+(2,6)



DIURNAL EXPANSION FUNCTIONS
1.0
0.5
0
-0.5
-1.0
NORMALIZED AMPLITUDE
0
30
60
90
DEGREES LATITUDE

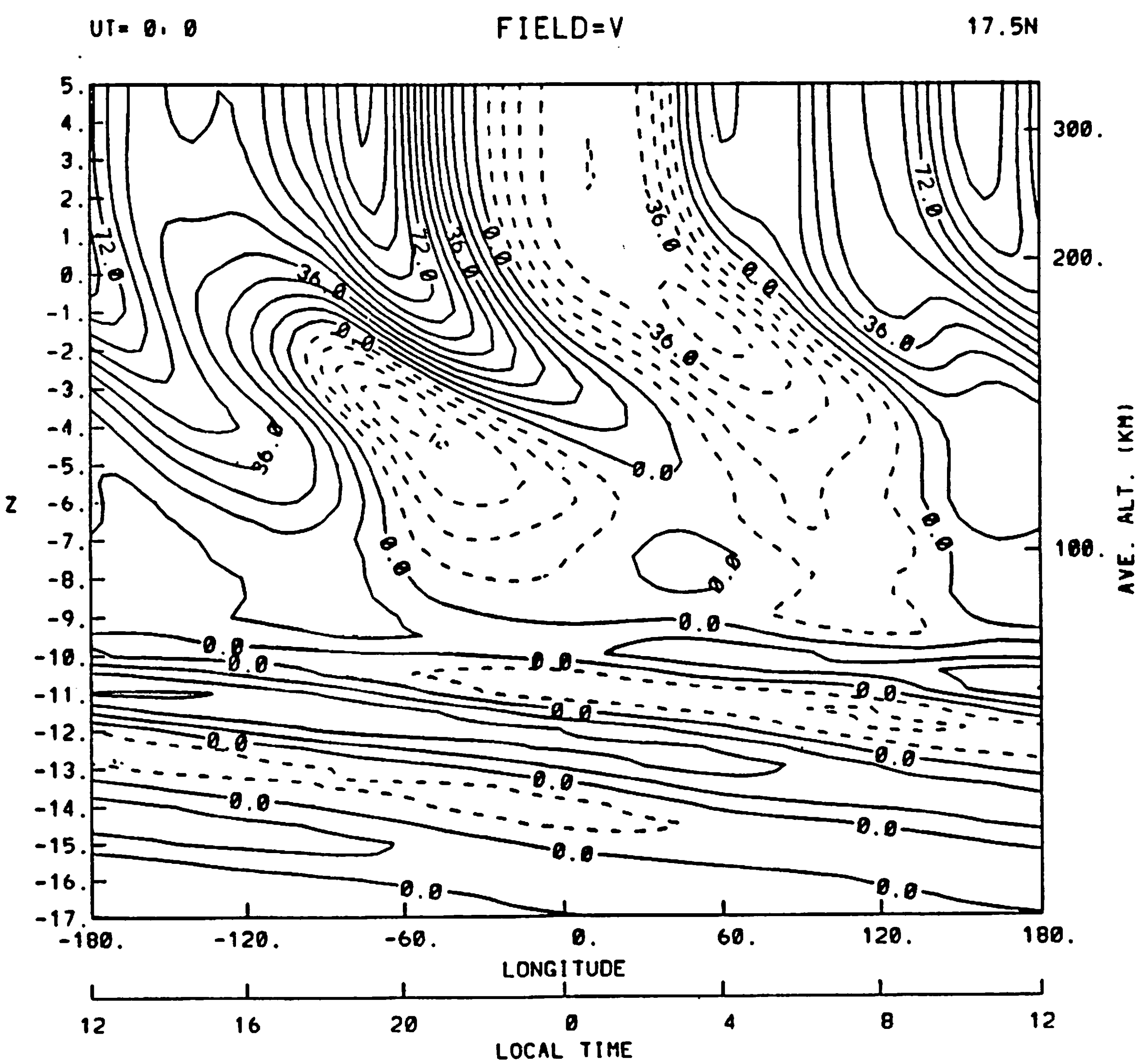

WESTERLY WIND
SOLAR SEMI-DIURNAL
EQUINOX
0 DEG. LATITUDE
18 DEG. LATITUDE
42 DEG. LATITUDE
60 DEG. LATITUDE
HEIGHT (KM)
AMPLITUDE
PHASE
0
40
80
120
160
200
240
280
320
360
400
0
2
4
6
8
10
12



WESTERLY WIND
SOLAR DIURNAL
EQUINOX
X 0 DEG. LATITUDE
⊙ 18 DEG. LATITUDE
✱ 42 DEG. LATITUDE
⊡ 60 DEG. LATITUDE
HEIGHT (KM)
AMPLITUDE
PHASE

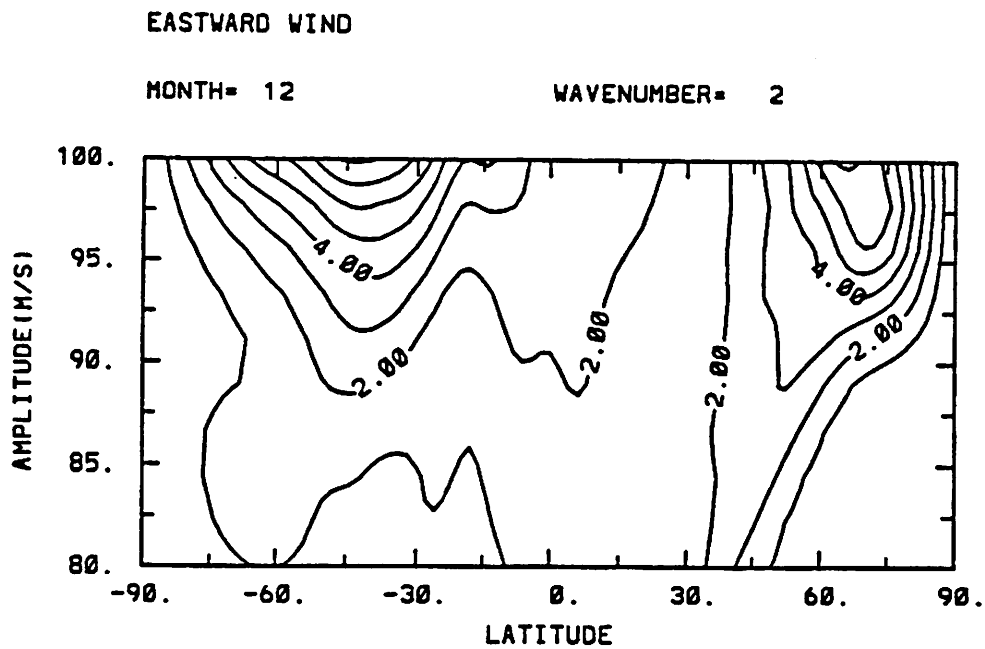
Thermosphere-Ionosphere-Mesosphere-Electrodynamics - GCM
TIME - GCM

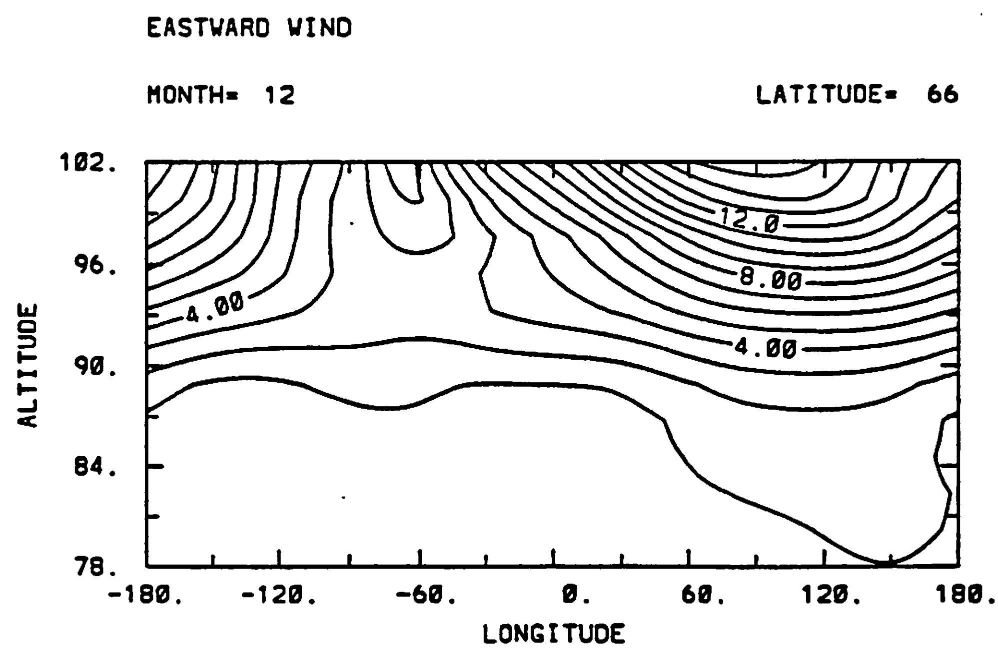
December Solstice



MONTHLY SIMULATIONS OF THE LUNAR SEMIDIURNAL TIDE*

(Migrating and Non-migrating)





*Vial and Forbes, JATP, 1993, submitted

Doppler-shifting effects of mean winds

For a tropospheric mean zonal wind $\overline{u} = \overline{U} \sin\theta$ (effectively a uniform superrotation of the atmosphere), then "σ", i.e.,

$$\frac{\partial}{\partial t} \rightarrow -i\sigma$$

should be replaced by the Doppler-shifted or intrinsic frequency, σ_D:

$$\frac{\partial}{\partial t} + \frac{\overline{u}}{a \sin\theta} \frac{\partial}{\partial \lambda} \rightarrow -i(\sigma - k\overline{U}) = -i\sigma_D$$

where $k = \frac{s}{a}$. To an observer on the ground, however,

$$\sigma_{obs} = \sigma_D + k\overline{U}$$

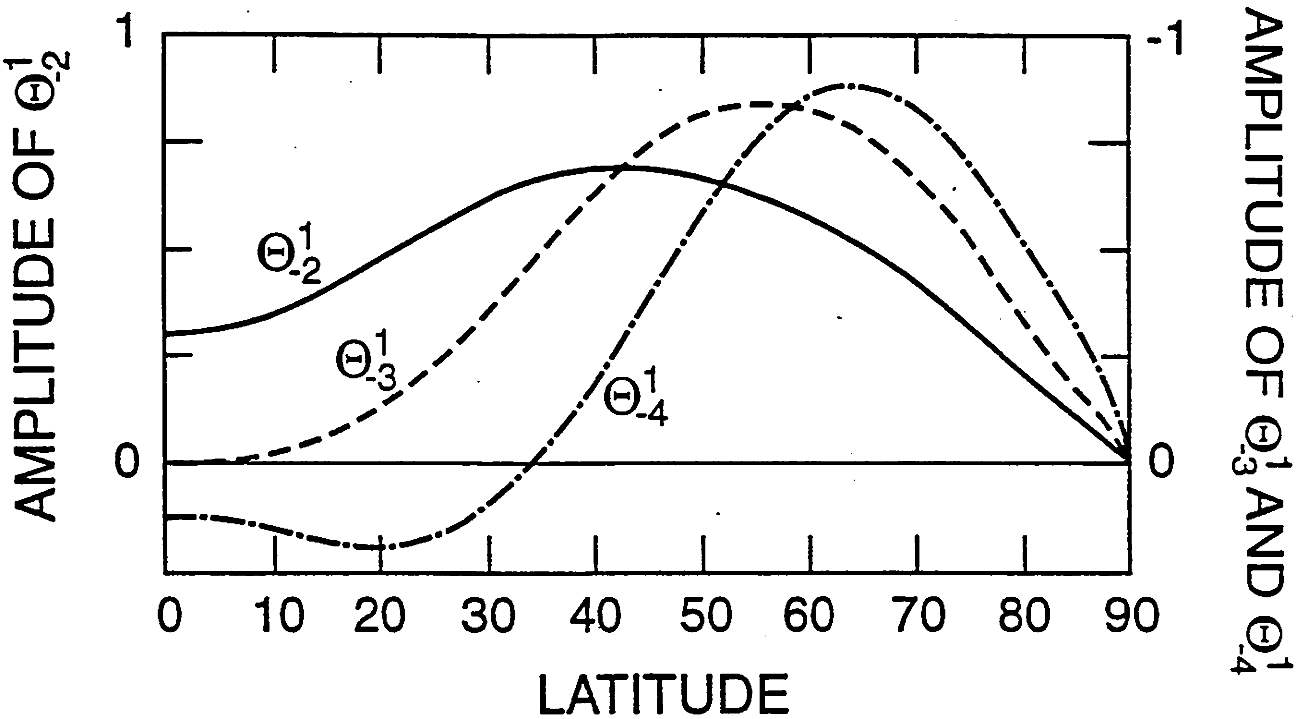
For the (1,-2), (1,-3), and (1,-4) normal modes:

T = 5 days → 5.6 days

T = 8.3 days → 10.2 days

T = 12.5 days → 17.1 days

Walterscheid [1980]



For a nonisothermal atmosphere:

(1) Above $\approx$ 90 km $\alpha^2 > 0$, implying propagating solutions and energy leakage into the thermosphere:

$$\alpha^2 = \frac{\kappa H + dH/dx}{h_n} - \frac{1}{4}$$

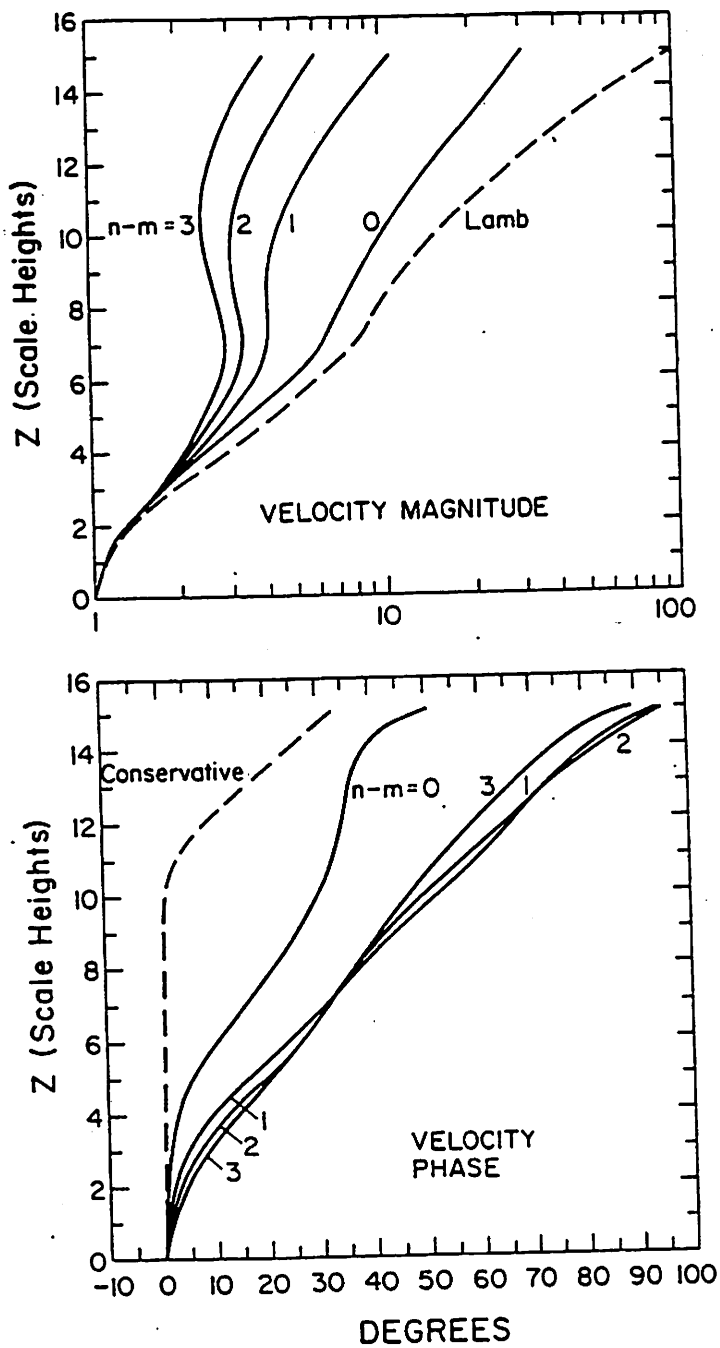
(2) A true resonance ("infinite response") no longer exists; a sharp maximum in the response exists *near* $h_n = \frac{H}{1-\kappa}$.

For a dissipative atmosphere:

(1) Amplitude of response decreases, and is less sharply defined.

(2) Vertical gradients of phase, vertical propagation, and vertical energy leakage are increased.

Salby [1980]



Z (Scale Heights)
n-m=3
2
1
0
Lamb
VELOCITY MAGNITUDE
0
2
4
6
8
10
12
14
16
1
10
100
Z (Scale Heights)
Conservative
n-m=0
3
1
2
1
2
3
VELOCITY
PHASE
-10
0
10
20
30
40
50
60
70
80
90
100
DEGREES

Propagation effects of mean winds

$$\frac{\partial}{\partial t} + \frac{\overline{u}}{a\sin\theta}\frac{\partial}{\partial\lambda} \rightarrow ik(-C_{ph} + \overline{u}) = -i\sigma_D$$

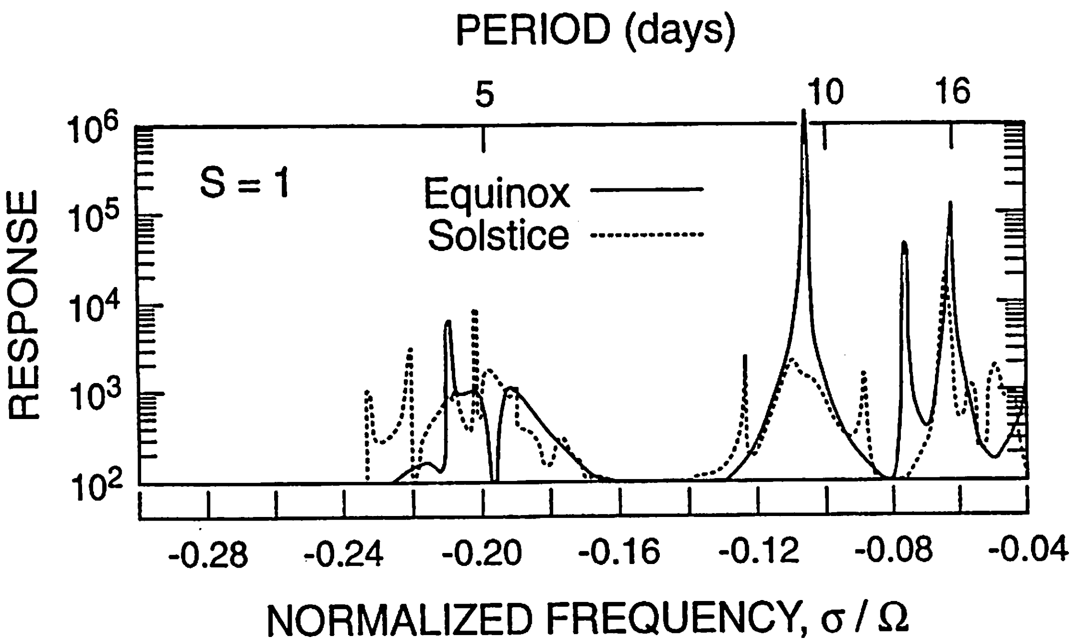
where $k = \frac{s}{a\sin\theta}$ and $C_{ph} = \frac{\sigma}{k}$ = the zonal phase speed. For westward-propagating waves,

$$C_{ph} = -\frac{\Omega a \sin\theta}{sT}$$

where T = period in days.

Wave	s	T	Latitude	C_{ph} (ms^{-1})
tide	1,2	$1,\frac{1}{2}$	0	464
tide	1,2	$1,\frac{1}{2}$	60	232
10-day	1	10	60	23
2-day	3	2	60	39

Salby [1981]



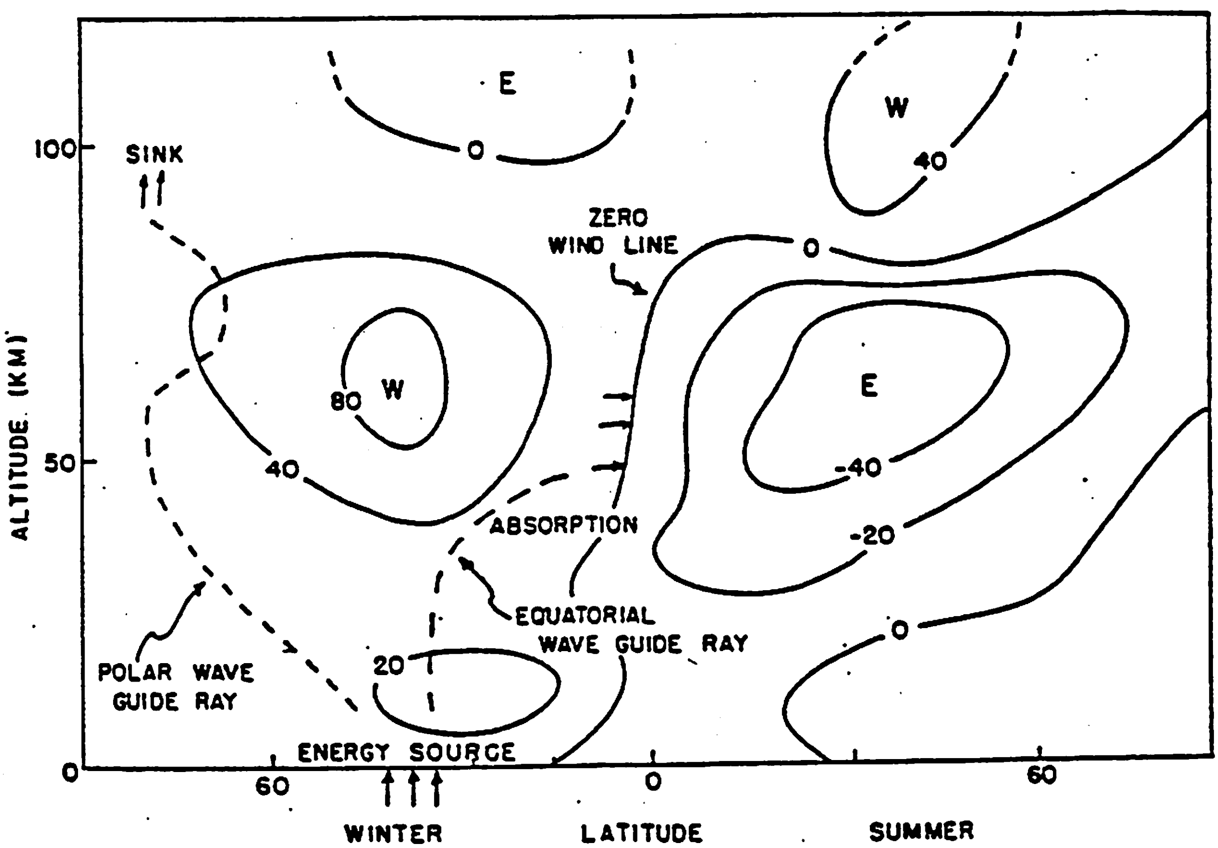
PERIOD (days)
5
10
16
10^6
10^5
10^4
10^3
10^2
S = 1
Equinox
Solstice
RESPONSE
-0.28
-0.24
-0.20
-0.16
-0.12
-0.08
-0.04
NORMALIZED FREQUENCY, σ / Ω

Propagation Inhibited

$$(\overline{U} - C_{ph}) < 0$$

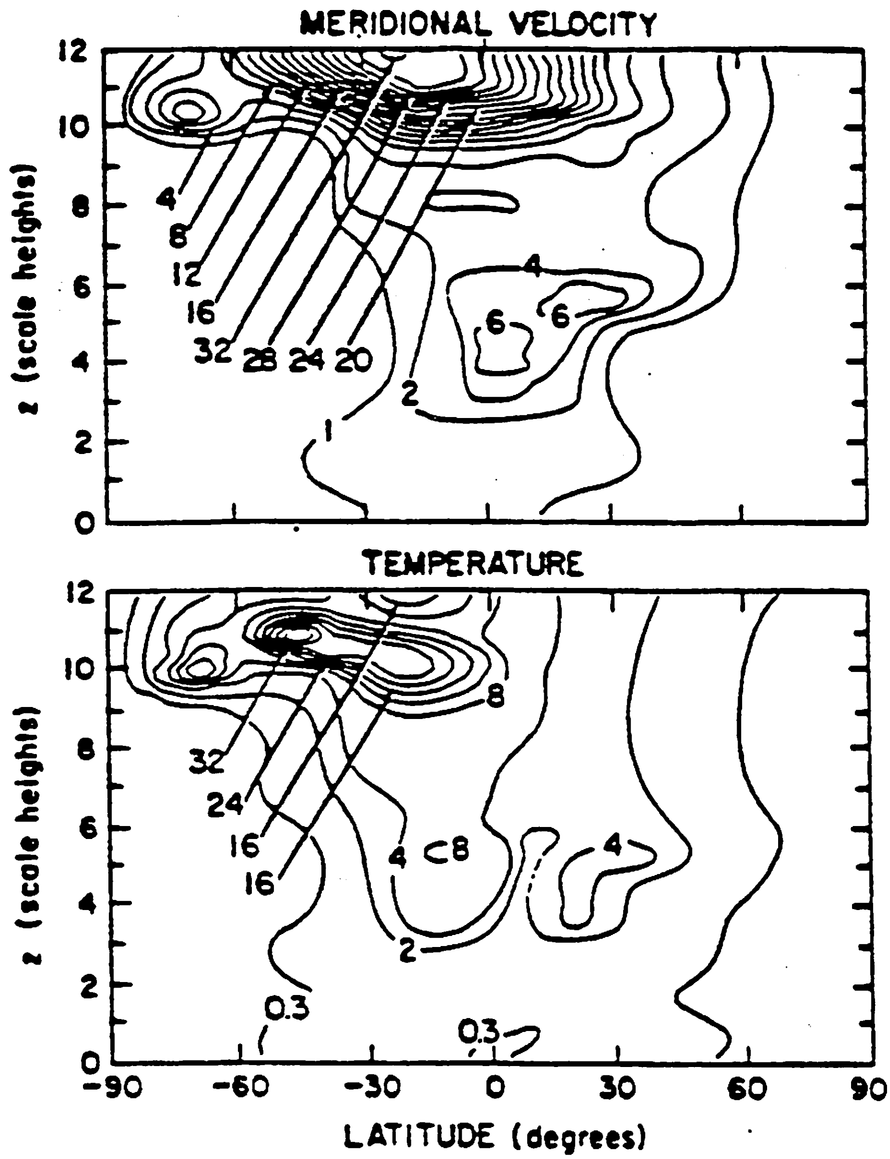
$$(\overline{U} - C_{ph}) >> 0$$

Dickinson [1968]

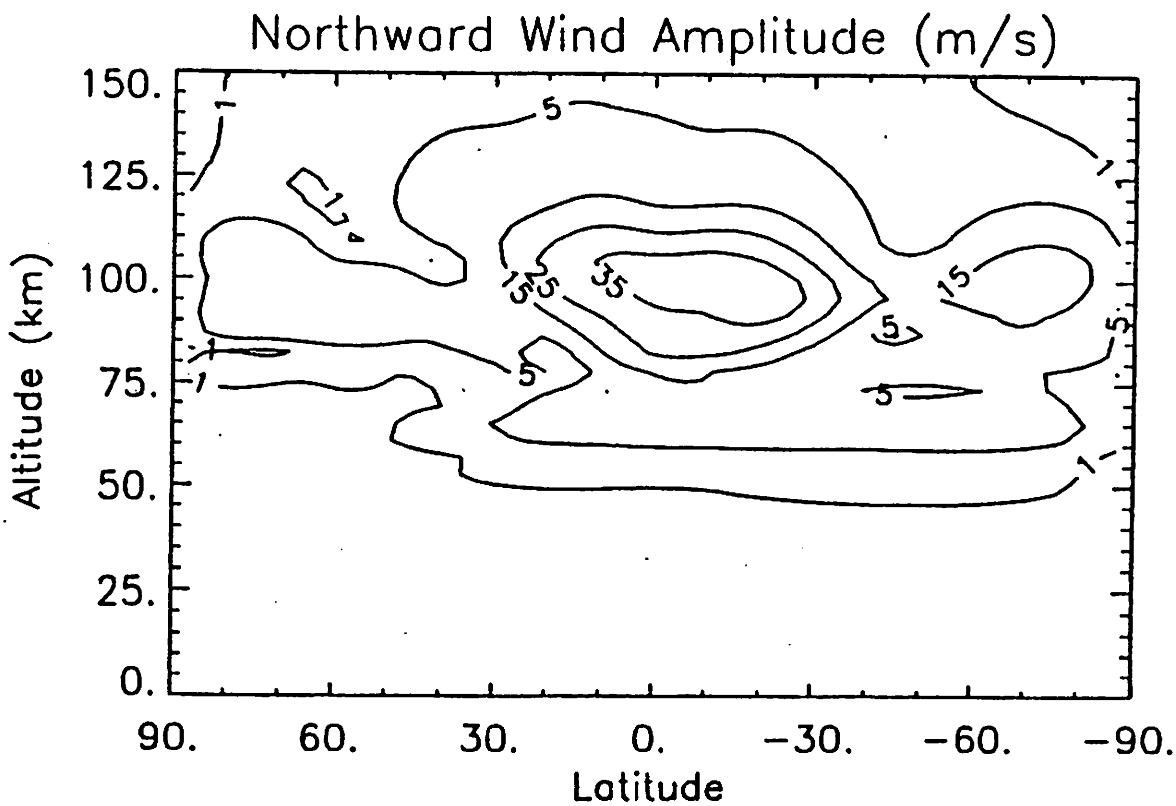


Simulated 2-day Wave

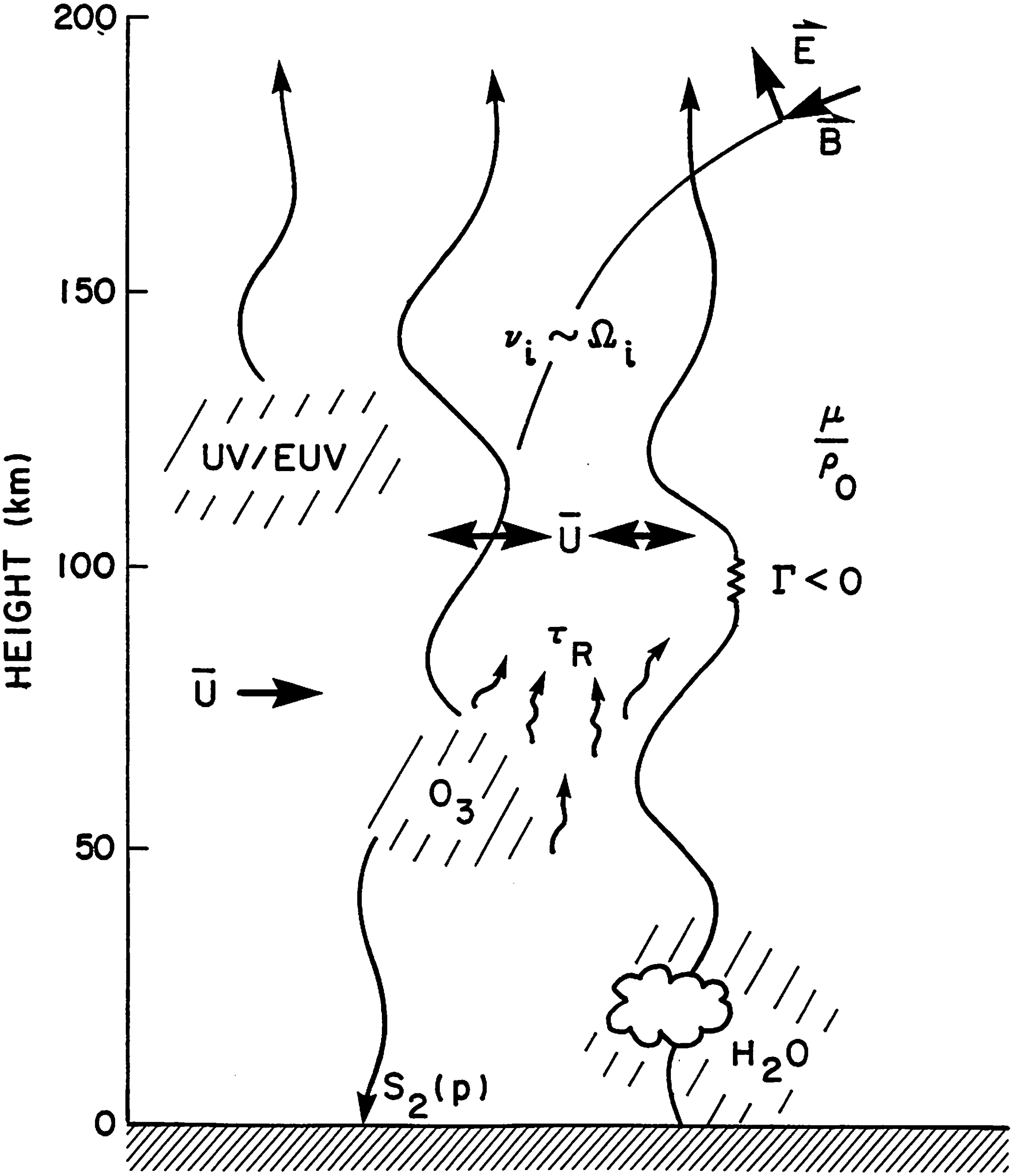
Salby [1981]



NUMERICAL SIMULATIONS
OF THE QUASI 2-DAY WAVE IN THE MLT REGION*



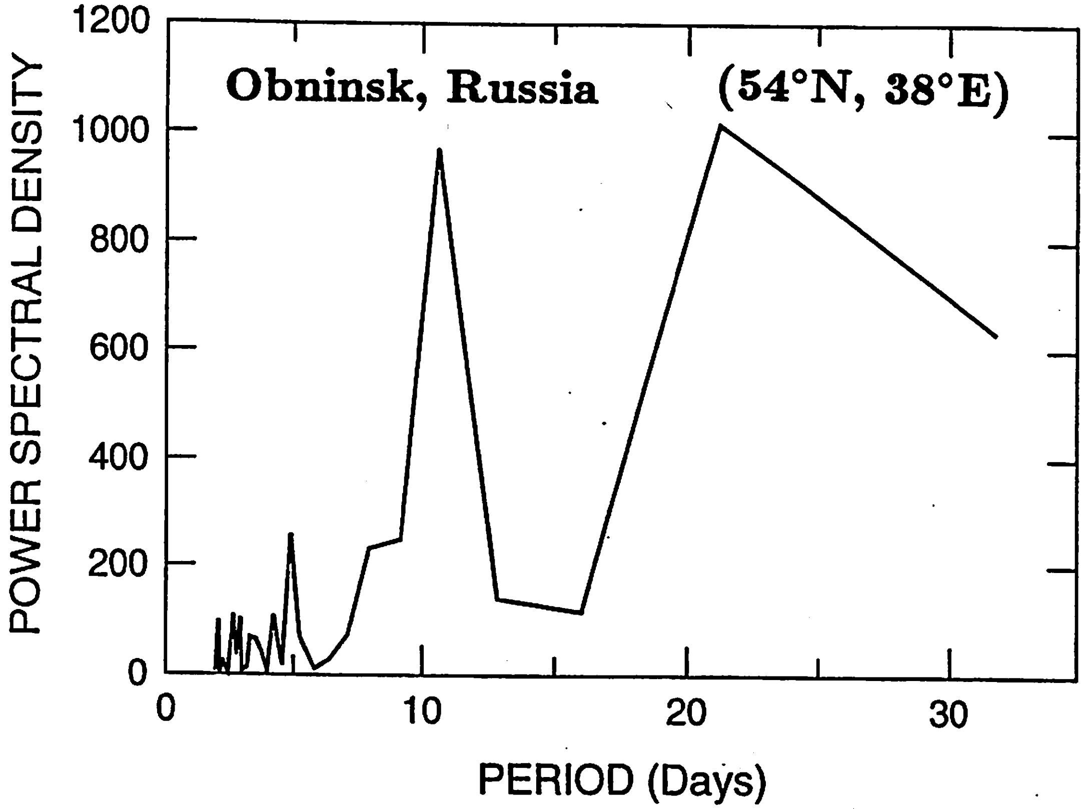
***Hagan et al., JGR, 1993, submitted**



TIDAL COUPLING OF ATMOSPHERIC REGIONS

Spectrum of Daily Semidiurnal Amplitudes

January-February, 1979



NON-LINEAR EFFECTS ON TIDAL AND PLANETARY WAVES IN THE LOWER THERMOSPHERE: PRELIMINARY RESULTS

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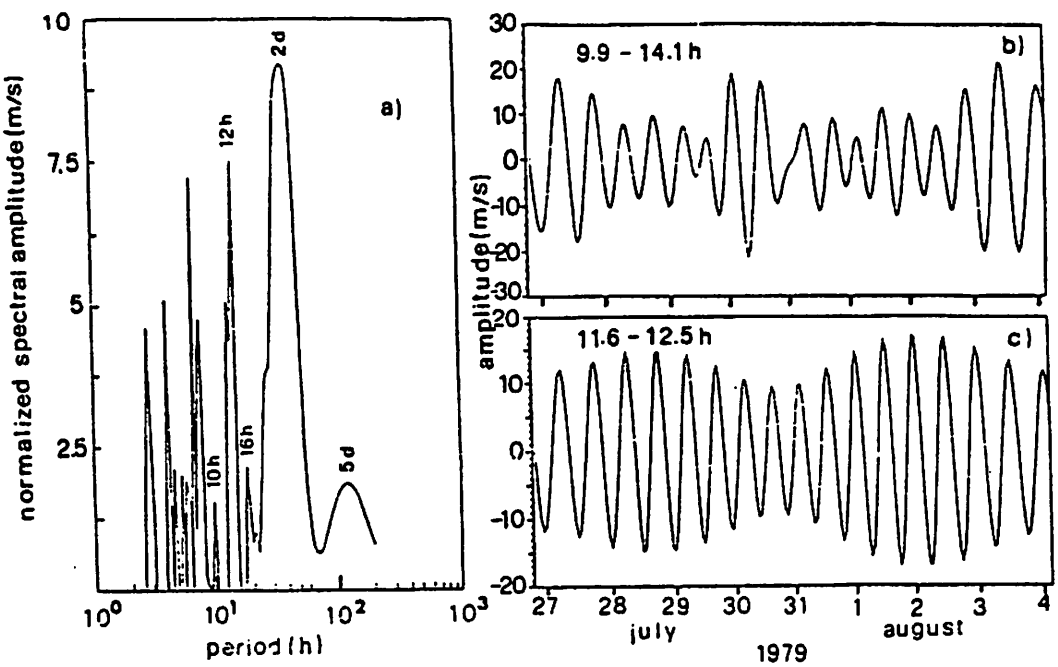

Fig. 1. a) Normalized amplitude spectrum of the zonal wind recorded at Bologna in the 90-100 km height region, for July 26 - August 4, 1979; b-c) Amplitude variations versus time of the semidiurnal tide at 90-100 km obtained through the observational period of a), by using the IFT method for two different temporal bandwidths in FT (b: 9.9 < T <14.1 h; c: 11.6 < T < 12.5 h). Time is relative to 20 h L.T., July 26, 1979.