Incoherent Scatter Radar 101
Joshua Semeter, Boston University (jis@bu.edu)
Phil Erickson, MIT Haystack Observatory (pje@haystack.mit.edu)

Prerequisites:
- Basic electromagnetic theory
- Basic concepts of analog signal processing
- Some plasma physics is helpful
- Logical and geometric thinking (well, pick one; very few of us can do both).

Syllabus:
- Frequency considerations.
- Radar Cross Section
  - Single electron
  - Volume of electrons
- SNR vs. more samples
- ISR spectrum: Nyquist approach
- How the spectrum is measured.
- Some auroral zone examples.

---

NSF Incoherent Scatter Radars

- Jicamarca, Peru
- Arecibo, Puerto Rico
- Sondrestrom, Greenland
- Millstone Hill, Massachusetts
- Advanced Modular ISR (AMISR)
What constrains the frequency of an incoherent scatter radar?

ISR operates at frequencies above plasma- and gyro-frequencies,

\[ \omega_p = \frac{N_e e^2}{\epsilon_0 m_e} \quad \omega_c = \frac{eB}{m} \]

but not so high that collective behavior of the plasma is lost. This means the wavelength must be greater than the Debye length

\[ \lambda_D = \frac{\epsilon_0 k_B T_e}{N_e e^2} \]

Thus the frequency must, therefore, satisfy

\[ \omega_p, \omega_c << \omega << \frac{2\pi c}{\lambda_D} \]

In this frequency range, each electron responds independently to the impinging field. The electrons respond independently (Born approximation), so the scattered signal is the superposition of the Doppler shifted scatter of each electron.

The physics is in the “Radar Cross Section” (RCS) denoted by \( \sigma \).
Radar cross section: single electron

\[ \sigma = 4\pi r_e^2 \text{power per unit area in the scattered wave at receiving antenna} \]

\[ = 4\pi \left| \frac{E_i(r_e)}{E_i(0)} \right|^2 \]

From classical EM:

\[ \left| \vec{E}_i(r_e, t) \right| = r_e \sin \theta \left| \vec{E}_i(0, t') \right| \]

where \( r_e = \frac{e^2 \mu_0}{4\pi m_e} = 2.82 \times 10^{-15} \text{[m]} \) is the classical electron radius.

Therefore we have \( \sigma_e = 4\pi (r_e \sin \theta)^2 \approx 10^{-28} \sin^2 \theta \text{[m}^2] \)

Arrays of radiators and the importance of phase

AMISR relies on constructive interference of individual elemental radiators to focus its beam. ISR relies on constructive interference of electron of dipole radiators in the direction of the receiving antenna.
RCS for volume distribution of electrons

Assume a plane-wave electric field incident on a scattering volume with magnitude $E_0$ assumed constant within the volume. Let us consider a monostatic radar ($\theta_0=\pi/2$). The backscattered field at the antenna due to an electron located at position $r_p$ in the volume is given by

$$E_{s,p}(t) = -\frac{r_p}{R} E_0 e^{j(k_0 r_p - \omega_0 t)}$$

where $k_0 = 2\pi/\lambda_{\text{radar}}$ is the wave number. The total field is the superposition of contributions from individual electrons:

$$E_s(t) = -\frac{r_p}{R} E_0 e^{j\omega_0 t} \sum_{p=1}^P e^{-j2k_0 r_p}$$

Note that, for the purposes of computing amplitude, the entire volume may be considered to be at a single average range $R$.

Of course we cannot make that approximation in the phase term!

---

RCS for volume distribution of electrons (continued)

We can rewrite the discrete sum as the integral over a differential quantity—the density. In general, the density varies in both space and time, so we have

$$E_s(t) = -\frac{r_p}{R} E_0 e^{j\omega_0 t} \sum_{p=1}^P e^{-j2k_0 r_p} = -\frac{r_p E_0}{R} e^{j\omega_0 t} \int N(r,t) e^{-j2k_0 r} d^3r$$

This expression is a 3D spatial Fourier transform, except that the argument of the exponential is twice the radar $k$-vector. If we let $k=2k_0$, we have

$$E_s(t) = -\frac{r_p}{R} E_0 e^{j\omega_0 t} \int N(r,t) e^{-jkr} d^3r$$

$$= -\frac{r_p}{R} E_0 e^{j\omega_0 t} N(k,t)$$

The magnitude of the scattered signal is proportional to the intensity of spatial fluctuations at $\frac{1}{2}$ the wavelength (twice the wavenumber) of the radar. Note: the mean value of the density is given by $N(k=0,t)$, but this corresponds to an infinite radar wavelength!

Incoherent scatter radars do not measure density directly. They measure fluctuations in density that match the wavelength of the radar.
RCS for volume distribution of electrons (continued)

In the literature, this is often made explicit by substituting $N$ with $\Delta N$ in the above, and using $N_0$ for “mean density”. With these variables, we can readily obtain

$$E_s(t) = -\frac{r_s E}{R} e^{j \omega t} \Delta N(k, t)$$

$$|E_s(t)|^2 = \left( \frac{r_s E}{R} \right)^2 |\Delta N(k, t)|^2$$

$$\sigma = 4 \pi R^2 \frac{|E_s(t)|^2}{|E_s(t)|^2} = 4 \pi R^2 |\Delta N(k, t)|^2$$

Next important observation: $\Delta N(k, t)$ is a random variable, so the instantaneous power (or, instantaneous cross section) is not useful. Rather we are interested in the mean, or expectation, value. Assuming time stationarity we obtain

$$\sigma = \langle |\Delta N(k, t)|^2 \rangle$$

Incoherent averaging

![Image showing incoherent averaging process with SNR=infiniti! and data for 10, 100, and 1000 pulses.]
SNR versus more samples

The essential result to this effect is from Farley, 1969:

\[ \frac{\Delta P_s}{P_s} = \frac{1}{\sqrt{K}} \left( 1 + \frac{P_n}{P_s} \right) \]

- \( P_s \) = signal power
- \( \Delta P_s \) = uncertainty in signal power
- \( K \) = number of samples
- \( P_n \) = noise power

Once you have high SNR (\( P_s/P_n >> 1 \)), all that matters is the number of independent measurements.

Sixto says:

A big radar helps, but size isn’t everything.

Differential cross section

We are not only interested the total power, but how this power is distributed in frequency. Let us now take the Fourier transform in time, noting that the time appears in two places on the right hand side.

\[ E_s(t) = \frac{r_e E_w}{R} e^{j\omega t} \Delta N(k,t) \quad \leftrightarrow \quad E_s(\omega + \omega_0) = \frac{r_e E_w}{R} \Delta N(k,\omega) \]

The effect of the density fluctuation spectrum appears as an offset to the radar frequency.

Using previous developments, we obtain the differential backscatter cross section (i.e., per unit frequency, per unit solid angle) at Doppler frequency \( \omega_0 + \omega \)

\[ \sigma(\omega_0 + \omega) d\omega = r_e^2 \left\langle |\Delta N(k,\omega)|^2 \right\rangle d\omega \]

(This is a general result, valid for many scattering problems)

Incoherent scatter radars measure fluctuations in density, not density itself. Are these two things related? Yes, …
What should \( \langle |\Delta N(k, \omega)|^2 \rangle \) to look like?

- Two essential theoretical approaches: microscopic and macroscopic.
- For once in plasma physics, particle approach and macroscopic approach agree exactly!
- Microscopic: Dressed test particle
- Macroscopic: Ion-acoustic wave spectrum.
  - Based on Nyquist dissipation theory
  - Radar picks one component of the thermal spectrum
  - Spectrum represents Landau-damped, Doppler-shifted scatter from this component.

ISR spectrum: Nyquist approach

- Send out a wave at \( \omega_o \) which corresponds to \( k = \omega_o / c \).
- The back-scattered signal is Doppler shifted by the forward and backward ion acoustic wave matching that \( k \)-vector (or wavelength).
- The backscattered signal has peaks at \( \omega_o + kC_s \) and \( \omega_o - kC_s \) where

\[
|C_s| = \frac{\omega}{|k|} = \sqrt{\frac{k_b(T_e + T_i)}{m_e + m_i}}
\]

- But not really: the wave modes are Landau damped!
The natural density fluctuations are ion acoustic modes. The collisionless ion acoustic wave has velocity:

\[ |C_s| = \frac{\omega}{|k|} = \sqrt{\frac{k_B (T_e + T_i)}{m_e + m_i}} \]

In the absence of wave damping, the backscattered power spectrum would look like this.

\[ |\omega_i| = |kC_s| = \frac{4\pi|C_s|}{\lambda_{\text{radar}}} \]

Landau damping affects the ion line because the ion thermal velocities are similar to the ion acoustic speed. The degree of damping is strongly affected by the ratio of electron and ion temperatures.
Primary information in the spectrum

- Ion temperature ($T_i$) to ion mass ($m_i$) ratio from the width of the spectra
- Electron to ion temperature ratio ($T_e/T_i$) from "peak_to_valley" ratio
- Electron (= ion) density from total area (corrected for temperatures)
- Ion velocity ($v_i$) from the Doppler shift

Courtesy C. Heinselman

Ion Temperature

Parameters
Freq: 449 MHz
$N_e$: $10^{12}$ m$^{-3}$
$T_e$: 2$^*$Ti
Comp: 100% O$^+$
n$_e$: $10^6$ kHz

Frequency (KHz)

<table>
<thead>
<tr>
<th>Lag (usec)</th>
<th>$T_i=500$</th>
<th>$T_i=1000$</th>
<th>$T_i=1500$</th>
<th>$T_i=2000$</th>
<th>$T_i=2500$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>50</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>100</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>150</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Courtesy C. Heinselman
Ion Velocity

 Parameters
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 1000 K
Te: 2000 K
Comp: 100% O^+
n_i: 10^6 KHz

Ion Mass

 Parameters
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 1500 K
Te: 3000 K
ν_i: 10^6 KHz

Courtesy C. Heinselman
Ion Composition (O$^+$ vs. H$^+$)

Parameters
Freq: 449 MHz
Ne: $10^{12}$ m$^{-3}$
Ti: 1500 K
Te: 3000 K
$n_\text{in}$: $10^{-6}$ KHz

How do we measure this spectrum?

Could use bank of filters, but not easy to make high-Q filters

Power spectrum and autocorrelation function are Fourier transform pairs and, so compute autocorrelation

$$R(\tau) = \int v(t)v^*(t-\tau)dt$$

$$R[j] = \sum_n x_n x_{n-j}^*$$

In other words, just complex multiplications of the return voltage samples
\( \tau_p \) = Length of RF pulse
\( \tau_s \) = Sample Period (typically \( \sim 1/10 \) pulse length)
Range-time diagram

Deriving ionospheric state parameters from spectrum

Ne, Te, Ti, Vi are derived by nonlinear fitting to the theoretical model

\[
\sigma(\omega) = \left| 1 + \frac{2}{4\pi^2} \sum \left( \frac{1}{D_j} \right)^4 F_{j}(\omega) \left| \frac{\partial \sigma}{\partial \omega} \right| + \frac{2}{4\pi^2 D_j} \sum \left( \frac{1}{D_j} \right)^4 F_{j}(\omega) \frac{\partial \sigma}{\partial \omega} \right|
\]

where:

\[
F_{j}(\omega) = 1 - \omega \int \exp \left( -\frac{16\pi^2 KT_i}{\hbar} \sigma r^2 \right) \sin(\omega \tau) d\tau
\]

\[
- j\omega \int \exp \left( -\frac{16\pi^2 KT_i}{\hbar} \sigma r^2 \right) \cos(\omega \tau) d\tau
\]

\[
F_{j}(\omega) = 1 - \omega \int \exp \left( -\frac{16\pi^2 KT_i}{\hbar} \sigma r^2 \right) \sin(\omega \tau) d\tau
\]

\[
- j\omega \int \exp \left( -\frac{16\pi^2 KT_i}{\hbar} \sigma r^2 \right) \cos(\omega \tau) d\tau
\]

Evans, 1969 IEEE review
Plasma parameters at auroral arc boundaries

Allsky images, 750 nm

Allsky camera images, 750 nm (N2 1PG), taken during the radar scan.

40 s scan, 48 km pulse
Single Pulse vs. Barker Code

40 s scan, 160 us pulse

2-min scan, 5-baud Barker code

Semeter et al., JGR 2005

3-dimensional ion motion

Vector Scale: 4000
Max Value: 4000
Error <= 250

2002 Feb 11
22:42:31.0 - 03:21:09.4

ehtspar 27949-27961 - Dec-5.4mng - taligma 032111-12-in-binmed.ps
All-sky image and horizontal ion velocity field overlain on global auroral image recorded by the IMAGE satellite. Inset plot shows vertical parameters (density, ion velocity, and net number flux).

Semeter et al., GRL, 2003