A long-term data set of globally averaged thermospheric total mass density

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We present a long-term data set, available on the CEDAR Data System, of globally averaged thermospheric total mass density derived from the orbits of ~5000 objects. The data cover the period 1967–2008 at heights from 200 to 600 km. The data have a temporal resolution of 3–6 days, a typical short-term precision of ±2%, and a long-term accuracy of 5–10%. We describe in detail the procedure used to generate the data set, provide an example of its scientific use, and discuss its limitations.

1. Introduction

Thermospheric total mass density is a key parameter for near-Earth space operations, and is also very useful for scientific studies of upper atmospheric behavior. Atmospheric drag exerts a significant influence on near-Earth orbits, and accurate predictions of orbital trajectories require precise knowledge of the ambient density and its spatial and temporal variations. Conversely, a satellite's trajectory contains information about the density of the medium it has traversed. The use of orbit data to infer thermospheric density has a long history [Sterne, 1958; King-Hele, 1966; 1987; Cefola et al., 2003; Storz et al., 2005; Picone et al., 2005]. Early density data sets derived from orbit data include those compiled by Jacchia and Slowey [1965; 1970; 1972; 1975] and Barlier et al. [1978]. Emmert et al. [2004] computed density values from 27 objects covering the period 1969–2001. Marcos et al. [2005] computed densities from 5 objects covering the period 1970–2000. Bowman et al. [2008] compiled 1978–2004 densities derived from numerous objects and employed this data set to develop improved empirical satellite drag models.

Currently, the two most commonly used methods for inferring density from orbit data are the Special Perturbations (SP) and General Perturbation (GP) approaches. In the SP approach [e.g., Bowman et al., 2004, 2008], a detailed numerical force model (the SP orbit propagator) is fit to tracking observations to derive a state vector and an effective drag parameter; density is then inferred from the drag parameter, which scales the atmospheric density model used with the SP propagator. In the GP approach, also known as the TLE approach, density is inferred from routinely compiled two-line orbital element sets (TLEs), using the change in the orbital period from one TLE to the next (see section 2). The 'GP' moniker refers to the use of an analytic force model (the GP propagator) in the production of the TLEs. The SP approach is more precise than the TLE approach, but the latter avoids the computational difficulties of reprocessing raw tracking observations to extract density. Picone et al. [2005] discussed in detail the relationship between the SP and TLE techniques.

Emmert et al. [2004] applied the TLE method to 27 long-lived objects to derive a long-term, negative trend in thermospheric density. Emmert et al. [2008] used the data described here to update and refine those results. Doornbos et al. [2008] used the TLE method to develop schemes for calibrating neutral density models. In this paper, we apply the TLE method to derive a time series of global average density.

The objective of the procedure described here is to produce a long-term data set suitable for studies of global thermospheric density variations on time scales greater than 3 days. By combining results from many objects into a single density database, the complications of analyzing densities from individual objects (with different and varying orbital paths) is removed. The use of a large number of objects also reduces the uncertainty in the final computed density. In our approach, most objects are debris with unknown properties. We therefore developed algorithms to identify objects unsuitable for density inference, and to empirically determine each object's ballistic coefficient (the parameter that relates density to the drag force). The algorithms and quality control procedures are ad hoc, but were uniformly applied and strictly adhered to.

The full procedure is described in the following sections, and consists of the following steps:

1) Initial object selection (section 2.2).
2) Estimation of ballistic coefficients (sections 2.3 and 2.4).
3) Computation of the average density, relative to a reference model, along the orbital trajectory of each object (section 2.4).
4) Combination of the density ratios from all the objects into the global average ratio, parameterized as a function of height and time (section 3).
5) Estimation of uncertainties in the ratios (section 4).
6) Computation of the absolute global average density (section 5).

Following the description of the procedure, in section 6 we present a scientific application of the data: the global response of thermospheric density to solar-cycle variations in extreme ultraviolet (EUV) irradiance. Finally, in section 7 we discuss the limitations of the data and in section 8 we summarize the procedure and the characteristics of the data set.
2. Computation of Density Ratios

2.1. Two-Line Element Sets

The U.S. military has routinely tracked space objects larger than 10 cm for several decades. Tracking observations have been distilled into two-line element sets (TLEs), which describe the average orbital state of each object within the tracking observation assimilation window, or fit span (typically 3 days). A TLE includes the following quantities: inclination of the orbital plane, right ascension of the ascending node (the northward equator crossing), eccentricity, argument of perigee (the angle between perigee and the ascending node), mean anomaly (the time since the last perigee, divided by the orbital period), and the orbital mean motion (the number of revolutions per day). The information contained in these six elements essentially corresponds to initial position and velocity vectors. The trajectory of an orbit can be computed from these elements and a compatible orbit propagator. The Simplified General Perturbations 4 (SGP4) propagator [Hoots and Roehrich, 1980] is compatible, and is suitable for computing low-precision (within ~1 km) trajectories. This precision is sufficient for our application: The trajectory is needed only for localizing the density information, and for computing corresponding predicted densities from empirical models.

The mean motion is the orbital element most directly relevant to the determination of density, since changes in the mean motion are directly proportional to the density along the trajectory; mean motion steadily increases as the orbit decays. There is no systematic inaccuracy in the mean motion, because it is essentially a timing measurement whose errors cannot accumulate. Changes in the mean motion are therefore also immune from systematic error.

TLEs are publicly distributed by Space-Track.org (www.space-track.org). The historical archive from 1967 to 2007 includes 64 million TLEs for over 29,000 objects.

2.2. Initial Object Selection

We first selected TLEs from the period January 1967 through September 2007. We restricted our search to TLEs with perigee heights between 100 and 600 km. The upper bound of 600 km avoids heights where the mass density of He is larger than that of O, and where solar radiation pressure effects may contribute significantly to changes in the orbital period. Also, our set of calibration objects (see section 2.3) does not adequately cover heights above 600 km. We included TLEs with perigee heights greater than 600 km only if the object dipped below 600 km at least once during the same calendar year. This selection criteria produced the sets {O1} and {T1} of objects and TLEs, respectively (see Table 1).

Table 1. Number of Objects and Data Records Within each Selection Set

<table>
<thead>
<tr>
<th>Set</th>
<th>No. of Objects</th>
<th>No. of TLEs or Density Ratios (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>{O1}, {T1}</td>
<td>20,171</td>
<td>14.69</td>
</tr>
<tr>
<td>{O2}, {T2}</td>
<td>8,307</td>
<td>11.53</td>
</tr>
<tr>
<td>{O3}, {R3}</td>
<td>5,827</td>
<td>6.11</td>
</tr>
<tr>
<td>{O4}, {R4}</td>
<td>5,341</td>
<td>5.47</td>
</tr>
</tbody>
</table>

We next analyzed the behavior of the mean motion time series for each object during each calendar year, and we developed several metrics designed to identify objects unsuitable for density calculations. The first metric is the annual average mean motion, calculated for each year; a minimum value of 6.4 orbits per day (orbital period less than 225 minutes; semi-major axis less than ~12,300 km) was required. This avoids the use of deep space objects (as defined by the U.S. Air Force [Vallado, 2001, p. 30]), whose orbits are significantly influenced by third-body gravitational perturbations.

The second metric is the 3-day monotonicity, M3, defined as the percentage of TLE pairs, separated by at least 3 days, whose mean motion difference is negative. Decreases in the mean motion indicate either a boost in the orbit (e.g., due to orbital maneuvers) or noise in the mean motion time series, and will produce negative density values. Negative density values are an unavoidable consequence of the statistical spread (due to limited precision) of mean motion changes and must be included when computing average density, to avoid skewing the results. However, we wished to avoid objects whose mean motion time series do not show consistent upward progress, and we imposed a minimum value of 75%.

The third metric is the drag-to-noise ratio (DNR), which serves a similar purpose to the signal-to-noise ratio. We first smoothed the mean motion to suppress variations with time scales less than 60 days, and computed the residuals around the smoothed time series. DNR is defined as the ratio of the standard deviation of the residuals divided by the mean motion, where the mean motion is a clear indication that the object was maneuvering. Decreases in the mean motion are therefore also immune from systematic error.

In addition to the restrictions on these three metrics, we required an object to have at least 30 TLEs within a given year, so that we could obtain a reliable assessment of the object's suitability within each year. We also excluded TLEs during the last 30 days of an object's lifetime, in order to avoid rapid orbital changes prior to reentry.

We used the smoothed mean motion to identify extreme outliers in the time series. Outliers can arise for example when radar observations are attributed to the wrong object. We discarded records with mean motion values greater than 5 standard deviations away from the smoothed mean, re-smoothed the remaining mean motion values, and repeated this procedure once.

Table 1 shows examples of the object selection process for the year 2006. The top panel shows an object (Air Force catalog number 00060) that easily qualifies. The next panel shows the mean motion for an object that was disqualified for insufficient monotonicity. In the third panel, an extreme outlier was identified and discarded, and the object easily qualified. The object in the bottom panel was disqualified for excessive noise; the saw tooth pattern of the mean motion is a clear indication that the object was maneuvering.

We conducted the object selection process separately for each calendar year, so that an object that was disqualified...
one year might be used in another (e.g., if it is no longer maneuvering). The process produced sets \{O2\} and \{T2\} of objects and TLEs, respectively (see Table 1).

### 2.3. Computation of Model-Dependent Ballistic Coefficients

For each of the objects in \{O2\}, we computed a time series of model-dependent ballistic coefficients, \( \hat{B}_t^M \). These values represent the inferred ballistic coefficient (\( C_d \rho / m \)) of the object, given a model density along the trajectory of the object and assuming that the true ballistic coefficient, \( B_t^T \), is constant during the period of evaluation. Following Picone et al. [2005] and Emmert et al. [2006a], we calculated the \( B_t^M \) values using the following equation:

\[
B_t^M(t) = \frac{2}{3} \frac{\mu^{2/3} [n_M(t)]^{1/3} \Delta_p n_M}{\int_{t_i}^{t_j} \rho^n v^3 F dt}
\]

(1)

where \( \mu = GM \) is the gravitational parameter, \( n_M \) is the Kozai mean motion of the orbit (obtained directly from the orbital elements), \( i \) and \( j \) are the indices of a pair of TLEs, \( \rho^n \) is the NRLMSISE-00 [Picone et al., 2002] model density along the object’s trajectory, \( v \) is the magnitude of the orbital velocity, and \( F \) is a dimensionless factor that accounts for co-rotating winds. The time \( t_{ij} = \frac{t_i + t_j}{2} \), where \( t_i \) and \( t_j \) are the start and end times, respectively. \( \Delta_p \) is the difference operator; i.e., the \( \hat{B}_t^M \) value minus the \( \hat{B}_t^M \) value of \( n_M \). We imposed a minimum integration time of three days in performing these computations; i.e., for each TLE \( i \), the second TLE \( j \) is the earliest one satisfying the condition \( t_j - t_i \geq 3 \) days. The time step for the integration in equation (1) was 1 minute. The analytic propagator appropriate to each TLE (SGP or SGP4 [Hoots and Roerrich, 1980]) was used to compute the orbital trajectory from the epoch of that TLE to that of the next available TLE (note that the interval \([t_i, t_j]\) could encompass several TLEs).

### 2.4. Estimation of true ballistic coefficients and further object selection

The ratio of the true weighted-average density along a trajectory to the corresponding model density is given by [Picone et al., 2005]

\[
\alpha_t^M(t) = \frac{\int_{t_i}^{t_j} \rho^n v^3 F dt}{\int_{t_i}^{t_j} \rho^n v^3 F dt} = \frac{B_t^M(t_j)}{B_t^T}
\]

(2)

where the superscript \( M \) on the density ratio \( \alpha_t^M \) denotes its dependence on the choice of density model. Expanding on the approach employed by Emmert et al. [2006a], we estimated the true ballistic coefficient for each object in the set \{O2\} as follows.

First, we calculated estimated \( B_t^T \) values, denoted \( \hat{B}_t^T \), for a set \{O2R\} of 74 objects, using the method described by Emmert et al. [2006a], which minimizes the variation of \( B_t^T \) ratios among pairs of objects. The set \{O2R\} includes all the objects used by Emmert et al. [2006a], plus additional objects (mostly spheres) that were not aloft during the period covered by the Emmert et al. [2006a] study. We used Starshine 1 as the primary reference object with a specified \( B_t^T \) of 0.009744 m²/kg (\( C_d = 2.1 \)).

Next, we repeated the procedure and computed \( \hat{B}_t^T \) values for the remaining objects, \{O2I\}, in \{O2\}, this time using the objects in \{O2R\} as references. Instead of using every possible pair of objects in the reduction, we paired the objects in \{O2I\} with the objects in \{O2R\}, but not with each other. Also, we performed the computation separately for each year, obtaining a series of annual \( \hat{B}_t^T \) values, denoted \( \hat{B}_t^T \) (where \( k \) represents different years), for each object in \{O2I\}.

We assume that the reference objects in \{O2R\} have constant \( B_t^T \) values, but this is not a reasonable assumption for the objects in \{O2I\}, whose area-to-mass ratio may have changed over time (e.g., due to outgassing, loss of components, or cessation of active attitude control). Since our objective is to produce a long-term data set for climatological studies, we must avoid objects with unstable ballistic coefficients, which could introduce artificial trends into the data. To identify objects with unstable ballistic coefficients, we analyzed the temporal behavior of the \( \hat{B}_t^T \) values for each object, normalized to the object’s overall average value:

\[
b_k = \frac{\hat{B}_t^T}{\overline{\hat{B}_t^T}}
\]

(3)

We computed metrics from \( b_k \) to determine whether an object qualified for further analysis. The metrics are given in Table 2. The selection criteria, which depend on the number of values in each series, are given in Table 3.

### Table 2. Metrics used to Identify Objects with Unstable Ballistic Coefficients

<table>
<thead>
<tr>
<th>Metric</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_b )</td>
<td>No. of values (years) in the series</td>
</tr>
<tr>
<td>( \sigma_b )</td>
<td>Std. Dev. of ( b_k )</td>
</tr>
<tr>
<td>( \text{trend}_b )</td>
<td>Linear trend of ( b_k )</td>
</tr>
<tr>
<td>( \text{range}_b = \text{trend}_b \cdot \sigma_b )</td>
<td>Range of ( b_k ), implied by trend</td>
</tr>
<tr>
<td>( \text{σ}_\text{trend} )</td>
<td>Uncertainty of linear trend</td>
</tr>
<tr>
<td>( \text{σ}_\text{grend} =</td>
<td>\text{trend}</td>
</tr>
<tr>
<td>( \text{Significance of linear trend} )</td>
<td></td>
</tr>
</tbody>
</table>

### Table 3. Ballistic Coefficient Stability Selection Criteria

<table>
<thead>
<tr>
<th>( n_b )</th>
<th>( \sigma_b )</th>
<th>( \text{trend}_b )</th>
<th>( \text{range}_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>None</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>2</td>
<td>( \leq 10% )</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>3–5</td>
<td>( \leq 10% )</td>
<td>None</td>
<td>( \leq 12% ) or ( \sigma_\text{trend} \leq 2 )</td>
</tr>
<tr>
<td>6–10</td>
<td>( \leq 10% )</td>
<td>None</td>
<td>( \leq 10% ) or ( \sigma_\text{trend} \leq 2 )</td>
</tr>
<tr>
<td>11–20</td>
<td>( \leq 8% )</td>
<td>None</td>
<td>( \leq 10% ) or ( \sigma_\text{trend} \leq 2 )</td>
</tr>
<tr>
<td>( \geq 20 )</td>
<td>( \leq 8% )</td>
<td>( \leq 5% ) per decade</td>
<td>None</td>
</tr>
<tr>
<td>or ( \sigma_\text{trend} \leq 2 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 2 shows examples of \( b_k \) values as a function of year. The objects with blue circles were judged by the algorithm to have sufficiently stable ballistic coefficients; objects with red X’s were rejected. Object 09815 was rejected due to a large and significant linear trend (-7.3% per decade). Objects 21965 and 24721 had excessive standard
deviations (8.0% and 11.3%, respectively), and objects 24721 and 09883 had excessive trend ranges (-22.7% and -16.5%).

We computed a single $B^T$ estimate for each object by averaging the yearly values:

$$B^T = \{ \tilde{B}_i^T \}$$ (4)

We then applied these estimates to equation (2) to obtain density ratios:

$$\alpha_{i,j}(t_i) = \frac{B^T_j(t_i)}{\tilde{B}_j^T}$$ (5)

Here we have altered the notation somewhat; the subscript $j$ refers to the different objects, and $i$ refers to the different epochs in the time series for each object (the average of the two epochs used in the drag computation of equation (1)). Note that $\alpha_{i,j}$ does not represent a matrix, because the epochs $t_i$ are different for each object. We have also dropped the $M$ superscript from $\alpha$; henceforth $\alpha$ is understood to represent the ratio relative to NRLMSISE-00.

We assigned drag-weighted average height [Emmert et al., 2004] as the representative altitude for each ratio. Retaining only those density ratio records with heights between 150 and 600 km, we obtained the set $\{R_3\}$ of ratios and the set $\{O_3\}$ of objects (Table 1).

3. Processing of Density Ratios

3.1. Averaging and Smoothing of Density Ratios

We next combined the density ratios from set $\{R_3\}$ to obtain the average ratio as a function of time and height. Figure 3 shows an example of binned average ratios for the year 1989. The temporal bins are 2 days wide at 1-day intervals, and the altitude bins are indicated at the right of the plot, along with the average number of objects represented by each bin. The time series for the different altitude bins (and hence independent collections of objects) show a high degree of coherence, suggesting that averaging the results from different objects is very effective at removing variations unassociated with thermospheric density.

The increasing amplitude of the variations with increasing height indicates that they are in part due to variations of the true thermospheric temperature around that predicted by NRLMSISE-00.

The high degree of consistency among different height bins suggests that a smooth representation of the ratios can be obtained without significant loss of information, while at the same time interpolating over data gaps and reducing the influence of data that clearly does not fit within the pattern. To smooth the data as a function of height, we fitted the ratios to cubic splines with a node spacing of 150 km. The fit was constrained at the lower and upper bounds (150 and 600 km) to have a second height derivative of zero. Figure 4 shows examples of binned averages of ratios as a function of height, and the corresponding height fits.

The primary purpose of the height representation is to smooth and interpolate the ratios; the physical significance of the height dependence is discussed in section 7.

To produce a fully interpolated representation of the average ratios as a function of height and time, we fitted the data to cubic splines in both these variables. We used 3-day node spacing for the temporal smoothing. The left panel of Figure 5 shows an example of the binned averages and smooth fit for 1976, a solar minimum year in which the ratios show large statistical variability at the higher altitudes.

The blue curve shows the smoothed average density ratio as a function of time. Because the average ratio represents collections of orbits that sample the entire globe, it can be interpreted, with some assumptions, as the ratio of global average density to that predicted by NRLMSISE-00, as described in section 5.

3.2. Residual analysis, quality control, and final object selection

The binned average ratios shown in the left panel of Figure 5 are quite noisy, and do not exhibit the same consistency among different height bins that is typical for other years. We therefore developed a third and final selection algorithm to exclude objects and individual ratio values that appear to be statistically anomalous. The residuals of the data around the initial parameterized fit formed the basis of our quality control analysis. Figure 6 shows representative residual distributions for several objects and years. In most cases, the residual distribution appears to be Gaussian, but in many cases the distribution is skewed, bimodal, or has a mean significantly different from zero. Any of these conditions can influence the multi-object mean. As with the mean motion and ballistic coefficient algorithms (sections 2.2 and 2.4), we developed a set of ad hoc metrics to cull unsuitable objects.

As an initial quality control step, we computed the standard deviation, $\sigma$, and mean of the residuals for each object and year, and excluded residuals greater than $3\sigma$ away from the mean. We repeated this step (recalculating the standard deviation each time) until all residuals for the object were within $3\sigma$ of the mean. This procedure was intended to identify and exclude extreme outliers. For a sample of 350 ratios (the typical annual number for an object during later years), each iteration of the procedure would be expected to improperly exclude one residual that is part of the Gaussian population. However, even in these cases only one or two iterations were usually required, and the impact on the shapes of the final distributions is negligible.

The first metric is $\mu_{\alpha}^j$, the residual mean for each object, normalized to the standard deviation of the means for all objects within a given year:

$$\mu_j = \frac{\sum_{i=1}^{N_{rat}} (\alpha_{i,j} - \alpha_{i,j}^{fit})}{N_{rat}}$$ (6)

$$\mu_{\alpha}^j = \mu_j \left( \frac{N_{obj} - 1}{\sum_{j=1}^{N_{obj}} (\mu_j - \langle \mu_j \rangle)^2} \right)^{1/2}$$

where $\alpha_{i,j}$ is a ratio value for object $j$ at epoch $t_i$ (see equation (5)), $\alpha_{i,j}^{fit}$ is the corresponding fitted value, $N_{rat}$ is the number of ratios for object $j$ during the year, and $N_{obj}$ is the
number of objects used for that year. This metric represents the bias of the density ratios from an object, relative to the multi-object mean, and reflects the extent to which our overall \(B^T\) estimate for an object is valid within the given year. We excluded objects for which \(\mu^*_{ij} > 3\).

The second metric is \(\sigma^*_j\), the standard deviation of residuals for each object, normalized as follows. We first computed the overall (all objects combined) standard deviation in 4 height bins: 150–300 km, 300–400 km, 400–500 km, and 500–600 km. We then computed a reference standard deviation by interpolating (with respect to height) among these values. For objects whose average height lies outside those of the binned standard deviations, we extrapolated the trend at either end, except that at the low end the reference standard deviation is constrained to be at least as large as that of the binned standard deviation in the lowest altitude bin. Figure 7 illustrates the application of this procedure for three selected years. The blue symbols show \(\sigma_j\), the standard deviation of residuals for individual objects. The red circles show the four binned standard deviations, and the red line shows the interpolated reference standard deviation \(\sigma^*(z_j)\). The metric \(\sigma^*_j\) is computed as

\[
\sigma^*_j = \frac{\sigma_j}{\sigma^*(z_j)}
\]

where \(z_j\) is the average height of object \(j\). In the final density calculation, we excluded objects for which \(\sigma^*_j > 2\). This selection limit is indicated by the red dotted line in Figure 7.

The third metric is designed to assess whether the residual distribution for an object is reasonably Gaussian. We first computed the linear correlation, \(r_j\), between the residual values and their expected Gaussian deviates, based on the quantile of the residual (its position in the ordered series of residuals). If the distribution is Gaussian, then a plot of these two variables will form a straight line. The panels on the right of Figure 6 show examples of such plots, which are known as quantile-normal plots [e.g., Hamilton, 1992, chapter 1], along with the computed correlations. Note that because the series of points is monotonic, the correlation values tend to be very close to one, even when the distribution is clearly non-Gaussian. However, it is possible to use the small variations of \(r\) to judge the quality of the distribution. To do this, we performed a Monte Carlo simulation to obtain the range of \(r\) values expected for different sample sizes. The result of this simulation is shown in the left panels of Figure 8. For each of 11 sample sizes, we ran 100 trials of randomly generated Gaussian deviates, computed the quantile-normal correlation \((r)\) for each sample, and transformed these values to \(\ln(1-r)\). The \(\ln(1-r)\) values show a linear trend with respect to the logarithm of the sample size, and a fairly uniform spread of values (bottom left panel). To obtain a lower limit, \(r_{\text{min}}\), on \(r\) values that might occur by chance, we computed the 2\(\sigma\) prediction interval of the \(\ln(1-r)\) trend (dashed line in Figure 8), transformed it back to a correlation value, and added a cushion of 0.01 (dotted lines in Figure 8). We excluded from further processing objects for which \(r_j < r_{\text{min}}(N_j)\), where \(N_j\) is the sample size for object \(j\).

The panels on the right of Figure 8 show the actual distribution of quantile-normal correlation values. The \(\ln(1-r)\) values show the same linear trend as the idealized values, but shifted upward somewhat. This motivated our adjustment of \(r_{\text{min}}\) by –0.01, in order to avoid excluding a large population of objects.

The objects shown in Figure 6 provide examples of the selection criteria for each metric. Object 20546 has a well behaved residual distribution, and easily made the cut. Object 27070 is a borderline case whose residual distribution is slightly skewed, but it was retained. Object 02208 is too skewed, and object 05096 is bimodal; both were excluded. Object 03019 failed the residual mean test, and object 12849 failed the normalized standard deviation test.

Figure 9 further illustrates the quality control procedure for several years and objects. The black lines show density ratios \(\alpha_{kj}\) as a function of time for a random sample of 100 selected objects with perigee heights near that of the highlighted object (red). The green line shows the ensemble average of all objects. The left panels show the raw results, and the right panels show the results after applying the quality control criteria. In the top panels, the residuals of object 05096 show a ~120-day oscillation that does not follow the ensemble average. This oscillation is responsible for the residuals’ bimodal distribution (see Figure 6), which resulted in the object’s disqualification. In the second row of panels, the density ratios from object 03019 closely follow the ensemble average, but show a large offset toward smaller values. The residuals of object 12849 are very noisy; removal of this object and other noisy objects greatly reduces the scatter of the data around the ensemble average. In the bottom row of Figure 9, object 27392 initially had a few outlying residuals. With the removal of these outliers, the object qualified for final selection; its density ratios closely follow the ensemble average.

The application of the quality control procedure rarely affected the smoothed average ratios significantly. However, in a few cases they improved the consistency of the final result. The right panel of Figure 5 shows the effect of the procedure for the year 1976, in comparison to the raw results shown in the left panel. The refined binned averages are more consistent among neighboring height bins, and the height-time fit is able to follow the variations in the averages more closely than in the raw case.

This stage of the process produced a set \{O4\} of 5341 objects and a set \{R4\} of 5.47 million density ratios (see Table 1), on which the final smoothed average is based.

4. Error Estimation

The overall long-term (≥ 3 years) accuracy of the smoothed density ratios is limited by the uncertainty in the ballistic coefficient of the primary reference object (Starshine 1). A higher (lower) ballistic coefficient will proportionally decrease (increase) the inferred density values. Furthermore, this error may also depend on the phase of the solar cycle, because drag coefficients are predicted to vary as the thermospheric composition and temperature changes with solar activity, whereas we have assumed a constant value for each object. Based on thermospheric composition representative of heights below 300 km, Moe et
al. [1995, Tables 3–5] obtained theoretical drag coefficients that varied by ±5–10% over a wide range of solar activity conditions. In contrast, Pardini et al. [2006, Figure 2] obtained theoretical $C_d$ values that showed less than ±1% variation over the solar cycle below 500 km, but a ±2–6% variation at heights between 500 and 600 km, due to substantial He contributions to the solar minimum composition. Quantitative experimental estimates of the composition effect are difficult to obtain from orbit data, and only a few on-orbit experiments of gas-surface interactions have addressed this issue [Moe et al., 1998]. Further study is needed, but based on the range of composition-dependent theoretical $C_d$ estimates, we assume that the long-term accuracy of our data is within ±10%.

On short time scales, we can obtain more concrete estimates of the uncertainty in our reduced data. The consistency of the average density ratios at different heights suggests that the relative density variations they represent are quite precise. We can quantify this precision based on the statistical spread of ratios from different objects. The standard error of the mean is usually computed for this purpose:

$$\hat{\sigma}_m = \frac{\hat{\sigma}}{\sqrt{N}} \quad (8)$$

where $\hat{\sigma}$ is the sample standard deviation and $N$ is the number of observations in the sample. A key assumption is that the observations are independent random variables, but in our case the residuals from a given object typically show strong autocorrelation over 1–10 days (Figure 10). This is not surprising given that many objects, particularly those at lower altitudes, produce relative variations that closely follow the ensemble average but are offset from it. To address this violation of independence when estimating the uncertainty of binned averages, we use

$$\hat{\sigma}_m^{bin} = \frac{\hat{\sigma}^{bin}}{\sqrt{N_{obj}}} \quad (9)$$

where $\hat{\sigma}^{bin}$ is the standard deviation of the data in the bin. $N_{obj}$ is the number of different objects in the bin and is intended as a measure of the effective number of independent measurements in the sample; this approach has also been used by Emmert et al. [2006b]. The estimated uncertainties shown in Figure 5 were calculated using equation (9).

To check that the data derived from different objects are indeed independent, we computed the correlation of residuals from pairs of objects during selected years, thereby forming an $N_{obj} \times N_{obj}$ sample correlation matrix. From this point of view, the residuals from each object compose a sample of a random variable. Figure 11 shows the distributions of the sample correlation matrix values for the selected years, along with the distributions that would be expected if the population correlation matrix were equal to the identity matrix. The observed sample correlations are only slightly higher than what is expected to occur by chance, which supports the assumption of zero covariance among data from different objects.

Our smoothed density ratios represent the average density continuously as a function of altitude and time, not a point average, but the same principles apply. The smooth function was computed by an ordinary least-squares fit of the data to localized functions (B splines) in altitude and time. With this type of inverse problem, the covariance matrix of the inferred model parameters (i.e., the generalized equivalent of $\hat{\sigma}_m^{bin}$) is usually calculated by assuming that the data have a covariance matrix of $\sigma^2 I$. That is, the data are assumed to be uncorrelated and to have a uniform variance of $\sigma^2$. With this assumption, the covariance of the model parameters is

$$\text{cov} \hat{\beta} = \sigma^2 (X^T X)^{-1} \quad (10)$$

where $X$ is the data kernel defined by the model $Y = X \hat{\beta} + \epsilon$, $\hat{\beta}$ is the vector of model parameters, $Y$ is the vector of data values, and $\epsilon$ is the vector of model residuals [Graybill, 1976]. For our smoothed density ratios, each column of $X$ contains the values, corresponding to the epochs and heights of observations $Y$, of one 2-dimensional B spline. In the case of a simple point average, $X$ is a vector of ones the same length as $Y$, $\hat{\beta}$ is a scalar equal to the sample mean, and equation (10) reduces to equation (8). For our application, it is clear that $\hat{\sigma}$ varies both with altitude and time, so this approach is not adequate. To accommodate variations in $\hat{\sigma}$, we fit the squared residuals as a function of height and time, similarly to the fit of the ratios, except that we used 10-day node spacing for the temporal splines. The plots in the first column of Figure 12 show examples of the fitted $\hat{\sigma}$, denoted $\hat{\sigma}^{fit}$, along with corresponding standard deviations of binned residuals.

To account for the autocorrelation of the residuals for each object, we assume, following Graybill [1976, section 6.8], that the residuals are normally distributed and their covariance is

$$\text{cov} \ = \sigma^2 V \quad (11)$$

where $V$ is an $N \times N$ matrix containing the scaled covariance of each pair of observations, and $\sigma^2$ is an overall variance parameter to be estimated from the data. We further assume that $V$ has the form

$$V_{ij} = \exp \left( -\frac{|t_i - t_j|}{\tau} \right) \quad (12)$$

if observations $i$ and $j$ are from the same object, and zero otherwise; $t_i$ and $t_j$ are the times of observations $i$ and $j$; and $\tau = 3$ days. Since $V_{ij} = 1$ in this case, $V$ is the Pearson correlation matrix of the residuals. The specific form of $V$ given in equation (12) is computationally convenient because, if the observations are time-ordered, the inverse of $V$ is a tri-diagonal matrix with

$$\text{cov} \hat{\beta} = \sigma^2 (X^T X)^{-1} \quad (10)$$
\[ \left( V^{-1} \right)_{ij} = \frac{1-V_{\text{e}ij}^{1}}{(1-V_{\text{e}i}^{1})(1-V_{\text{e}j}^{1})} \]

\[ (V^{-1})_{ij} = \frac{-V_{ij}}{1-V_{ij}} \quad (13) \]

\[ (V^{-1})_{ij} = (V^{-1})_{ij} \quad (14) \]

With this statistical model,

\[ \text{cov} \hat{\beta} = \sigma^2 \left( X^T V^{-1} X \right)^{-1} \quad (15) \]

where \( x \) is the vector of basis function values for the selected location. Comparing with equation (9), the quantity

\[ N_{\text{eff}} = \frac{1}{(X^T V^{-1} X)^{-1}} \quad (16) \]

can be viewed as the effective number of independent observations supporting the point estimate. In the case of the simple mean, \( N_{\text{eff}} = \sum_{i\ell} (V^{-1})_{ij} \), and if the observations are uncorrelated, \( V^{-1} = I \) and \( N_{\text{eff}} = N \). The second column of Figure 12 shows examples of \( N_{\text{eff}} \) derived from the density ratio times and heights. The number of different objects \( (N_{\text{obj}}) \) in 3 day x 150 km bins, centered on the indicated times and heights, is superimposed. As expected, \( N_{\text{eff}} \) and \( N_{\text{obj}} \) are very similar, provided the bin size used for \( N_{\text{obj}} \) is close to the node spacing of the basis splines used for \( N_{\text{eff}} \).

Based on the statistical model described above, we adopted the following quantity as estimate of the density ratio uncertainties:

\[ \hat{\sigma}^2_{\alpha}(t,z) = \frac{\hat{\sigma}^2_{\alpha}(t,z)}{N_{\text{eff}}(t,z)} \quad (17) \]

The last column of Figure 12 shows examples of \( \hat{\sigma}^2_{\alpha} \) values computed using equation (17) and corresponding binned values, \( \hat{\sigma}^2_{\text{bin}} \), computed using equation (9). The uncertainty is 1–3% for the more recent, data-rich years of 1996 and 2002. In the worst case of high-altitude, 1976 data, the uncertainty is 4–12%. In 1970, the uncertainty spikes near days 70 and 90 as a result of the absence of data around these days. Figure 13 shows fitted density ratios for the selected years and heights, with the equation (17) uncertainties depicted as the ±1σ interval around the mean. Except for the periods of large uncertainties in 1970 and 1976, the uncertainties are much smaller than the temporal variations in the mean.

We expect that the dominant source of the short-term uncertainty is random error in the mean motion values used to derive the density ratios. Other sources are variations in the ballistic coefficients (e.g., due to tumbling), solar radiation pressure influences on the orbits, and local time-latitude variations in the ratio of the true density to that of NRLMSISE-00 (see section 7). Over the large collection of objects, these sources of error should largely cancel out.

Our procedure for estimating the uncertainty of the mean is divorced from the procedure for estimating the mean itself, and there is consequently some inconsistency in the approach. The latter procedure employs ordinary least squares (OLS), whereas the former employs maximum likelihood (ML) and assumes some knowledge of the covariance and distribution function of the data. In a pure ML application, the data covariance \( \sigma^2 V \) provides weighting factors for the computation of both \( \hat{\beta} \) and \( \text{cov} \hat{\beta} \). Furthermore, equation (11) assumes that the residuals have uniform variance, whereas Figure 6 shows that the sample variances of residuals from different objects have a wide range of values. Despite these inconsistencies, our approach appears to provide reliable uncertainty estimates and achieves the primary goal of only treating observations from different objects as fully independent.

5. Computation of Global Average Density

Rigorously, our object-averaged density ratio time series represents ratios of weighted orbit-averaged true density to weighted orbit-averaged model density, averaged over a collection of different orbits. Applying the average ratios to the global average model density should provide a reasonable estimate of the true global average density at a given height (averaged over time scales of > 3 days). Essentially, we are assuming that the average of the ratios is very close to the ratio of the averages (of global true and model densities), as follows.

Suppose \( \{ \rho_j \} \) is a set of true densities averaged over different segments or areas \( j \) of the globe, such that the areas are all equal in size and evenly sample the entire globe, and let \( \{ \rho_j^M \} \) be corresponding model densities. The global average true and model densities are then

\[ P = \langle \rho_j \rangle \]

\[ P^M = \langle \rho_j^M \rangle \quad (18) \]

where the brackets denote the simple average of the enclosed set. We also define local and global density ratios as

\[ \alpha_j = \frac{\rho_j}{\rho_j^M} \]

\[ \Lambda = P/P^M \]

so that

\[ \Lambda = \frac{\langle \alpha_j \rho_j^M \rangle}{\langle \rho_j^M \rangle} = \frac{(\langle A + \Delta \alpha_j \rangle P^M + \Delta \rho_j^M)}{P^M} = \Lambda + \left( \Delta \alpha_j \frac{\Delta \rho_j^M}{P^M} \right) \quad (19) \]

where
The ratio of the true to modeled global average density thus equals the average of the local ratios plus a term that convolves the relative variations around the globe of the local ratios and model densities. Provided these two sets are not correlated, this term should average out close to zero. In reality, one might expect the global variations (i.e., latitudinal and local time variations) of the true density to be, to some extent, amplifications or reductions of the model density, which presumably is giving a reasonable estimate of global variations. In the worst case, the two terms would be perfectly correlated, such that

\[
\Delta \alpha_j = \frac{\Delta \rho^M_j}{\rho^M_j} = c \left( \frac{\Delta \rho^M_j}{\rho^M_j} \right)^2
\]

(22)

and the error induced by using the average local ratios is directly proportional to the variance of the model density around the global average model density.

Our actual orbit-derived density ratios for each object are analogous to \( \alpha \), and our smoothed object-averaged time series of ratios is analogous to \( A \). The ‘area’ (actually a line) sampled by each orbit, however, is not the same as the other orbits. Circular orbits sample circles, and elliptical orbits sample the orbit segment near perigee. (In this analogy, we view the orbit-derived densities as simple orbit-average densities and not as weighted averages; this should be a fair characterization of the data, since \( F \) and \( v \) in equation (1) are roughly constant for circular orbits and for the portion of elliptical orbits near perigee.) Averaging the local ratios represented by the individual orbits therefore provides a good estimate of the global ratio (and hence the global average true density, once the estimate of the global ratio is applied to the global average model density), provided: 1) The local ratios are not strongly correlated with the variation of the model density (i.e., \( c \) in equation (22) is small), OR 2) The variation of the local ratios (\( \Delta \alpha \)) is sufficiently small, OR 3) The variation of the local model density (\( \Delta \rho \)) is sufficiently small. Provisions 2 and 3 are helped in our case by the fact that the orbits, especially circular ones, already sample a significant portion of the globe, so that the variations of the density (either true or modeled) among different orbits should be reduced. Also, the fact that we are averaging over several days should reduce the error. Emmert et al. [2006a, Figure 1] showed that the density ratios tend to be very similar even when the perigee location is very different. Doornbos et al. [2008] found that the global term is by far the most important in spherical harmonic calibration schemes for empirical density models. Both of these results suggest that provision 2 is reasonably met.

A good estimate of the global average density also requires that the collection of orbits samples the globe sufficiently evenly to overcome any minor variations of the local density ratios. To assess the evenness of the sampling, we computed the contribution of individual orbital trajectories to the sampling of local time, latitude, and altitude. For a specified time interval, we generated a trajectory on an evenly spaced time grid for each object. Each grid point of each object was assigned a contribution score equal to the NRLMSISE-00 density at that location and time divided by the total NRLMSISE-00 density along the trajectory. The total score is thus one for each object; the scores of elliptical orbits are concentrated near perigee, while those of circular orbits are more evenly distributed.

We then binned the scores for all the objects by altitude, local time, and latitude, and computed an average for each bin. The average was then adjusted to a sampling density by dividing by the surface area of the bin, to obtain the average score per steradian. Figure 14 shows examples of the relative sampling density, during days 270–273 of 1970 and 2002, as a function of local time and latitude for 3 different height bins. At the lower altitudes (200-300 km), sampling is somewhat discrete but fairly evenly distributed on global scales, except that lower latitudes are overrepresented. At the higher altitudes, the coverage is much better and more even, although in general the higher latitudes are slightly overrepresented (presumably due to the large number of circular polar orbits; even a single polar orbit oversamples high latitudes). In the 1970 example, however, there is a sampling concentration near midnight at 30°S. This cluster is due to debris objects from two Soviet anti-satellite tests conducted in 1968.

To produce absolute global average densities from the time series of smooth ratios at a given altitude, we first computed the NRLMSISE-00 density every 3 hours on a 10° latitude x 60° longitude grid, in storm time mode with all switches on (the same configuration used in computing the ratios). We then averaged the density over latitude and longitude, weighting by the area subtended by each grid cell, and smoothed the global average using the same 3-day cubic B splines employed for the average ratios. Finally, we multiplied the smoothed NRLMSISE-00 density by the fitted ratios to obtain the absolute density. This computation is repeated at each altitude for which absolute densities are desired. Figure 15 shows the resulting absolute orbit-derived global average density during 1989 at three selected heights, along with the corresponding NRLMSISE-00 density.


We developed the density data set to study long-term trends in the thermosphere, as well as global variations associated with solar activity, geomagnetic activity (on time scales > 3 days), and seasonal oscillations. Emmert et al. [2008] presented long-term trends derived from the data. To further demonstrate the utility of the data set for studies of the thermospheric response to solar activity, we briefly assess the solar cycle dependence of the density data. We focus here on a comparison between the data's statistical dependence on two EUV proxies: the 10.7 cm radio flux, \( F_{10.7} \), and the 26–34 nm portion of the NRL solar spectral irradiance empirical model (NRLSSI). The latter is based on TIMED/SEE irradiance measurements and uses the \( F_{10.7} \) and Mg II indices as input (J. L. Lean et al., manuscript in preparation). The specific relationships between the two
indices and the observed irradiance variations are determined from multiple linear regression of both indices with the SEE observations, on rotational time scales. For most of the EUV spectrum (at wavelengths longer than about 25 nm) the Mg II index is a better proxy for short-term EUV irradiance variations than $F_{10.7}$ [Viereck et al., 2001]. The top panel of Figure 16 shows the 1978–2006 density data as a function of the NRLSSI 26–34 nm flux, denoted as EUVSSI. The symbols with error bars show binned averages of the data, and the solid line is a fit of the data to cubic splines in EUVSSI, with nodes at 0.79, 0.82, 0.85, 0.92, 1.09, 1.32, and 1.76 mW/m²; the fit is constrained to have zero curvature at the upper and lower bounds. The middle panel is the same, except that the data are plotted, binned, and fitted as a function of $F_{10.7}$. In this case, the nodes are the same as those used by Emmert et al. [2008]: 60, 70, 80, 100, 150, 220, and 350.

The density varies very smoothly as a function of both EUVSSI and $F_{10.7}$. The dependence on EUVSSI is noticeably flatter than the dependence on $F_{10.7}$, which suggests that EUVSSI is a more favorable index than $F_{10.7}$ for climatological modeling of density. The variability of the data around the EUVSSI fit is slightly smaller than around the $F_{10.7}$ fit; this reduction in variance is quantified in the bottom panel of Figure 16. The root-variance of the data around the EUVSSI fit is generally about 4% smaller (relative to the density, not the variance) than that of the data around the $F_{10.7}$ fit.

It is not new or surprising that a better representation the solar EUV flux results in better prediction of short-term density variations; similar findings were obtained, for example, by Lean et al. [2006] and Guo et al. [2008]. However, the temporal extent of the density data described in this paper enables a deeper statistical characterization of the relationships between thermospheric density and solar irradiance proxies, on both short and long time scales. We will explore these relationships in more detail in a future paper.

7. Criticism and limitations of data set

The temporal resolution of the data is 3 days at best, because we have smoothed the results with 3-day B splines. The temporal resolution is also limited by the minimum time difference (3 days) imposed on the TLE pairs used to compute the integrated density (equations (1) and (2)). Figure 17 shows the average integration spans as a function of year for different height ranges. The average span is as much as 11 days before 1970, less than 6 days after 1970, and 3–4 days after 1990. Finally, the mean motion values themselves represent an average value over the temporal span of the tracking observations from which they are derived. The fit span for TLEs is usually 3 days, but can be longer, particularly for the early observations; this information is not retrievable from the TLEs. It is not clear how these three different limits on the temporal resolution combine to determine the final temporal resolution, but inspection of the coherent variations seen in the binned ratio averages (such as in Figure 3) suggests that the resolution is about 4 days after 1970, and perhaps 6–8 days during 1967–1970. On these time scales, variations of the inferred densities could be damped somewhat by the influence of objects with coarser temporal resolution.

Neglecting the effect of composition on the ballistic coefficients may introduce some solar-cycle dependent bias into the inferred densities, as discussed in section 4, and may also produce a height-dependent bias. Systematic height dependent biases may also arise from the different global sampling characteristics at high and low altitudes: As mentioned in section 5, low-perigee (< 300 km) orbits tend to be more equatorial, whereas the collection of higher-perigee (300–600 km) orbits slightly oversamples high latitudes. The uneven sampling is greatly mitigated by the use of NRLMSISE-00 to estimate the local time and latitude dependence of the density, but may still produce small (perhaps less than 5%) height-dependent biases.

Because of these potential height-dependent biases, the absolute height dependence of the data should be treated with caution. However, the height dependence of derived parameters, such as the semiannual variation, should be reliable. The absolute height dependence of the global average density is perhaps not a useful parameter in any case. It cannot, for example, be used straightforwardly to infer the global average temperature. This is because the global average temperature is inversely proportional to the vertical gradient of the global average log-density (assuming hydrostatic equilibrium), not the global average density represented by the data set.

In future development, any bias introduced by the assumption of constant ballistic coefficients could be alleviated by allowing them to vary with ambient conditions, provided reliable estimates of this variation are available. The global sampling could be evened out by giving more weight to underrepresented regions; the weighting scheme would have to change as a function of time. Alternatively, the variation of density with local time and latitude could be assimilated along with the global mean, following Doornbos et al. [2008]. While this approach is straightforward for elliptical orbits, a technique for inferring spatial variations from circular orbits has not been developed. Restricting the assimilation to elliptical orbits would greatly reduce the number of available objects, and hence the quantity of information utilized.

8. Summary

We developed a long-term database of global average total mass density from the orbits of ~5000 objects. The density values were derived from routine orbital elements compiled by the U.S. Air Force. The data cover the years 1967–2007 and altitudes of 200–600 km. The first step in the procedure was initial object selection, in which all objects affected primarily by atmospheric drag (i.e., not maneuvering) were empirically identified and selected for further processing. The next step was empirical determination of the ballistic coefficient of each object, based on a set of known reference objects. We then excluded objects whose ballistic coefficients did not exhibit long-term stability. From the orbits of each of the remaining objects, we inferred a time series of density relative to the NRLMSISE-00 empirical model. The density ratios from the different objects were combined to produce the global average density ratio,
parameterized as a function of height and time, from which the global average density was computed.

The data are suitable for studies of global thermospheric variations and trends on time scales greater than a few days, and for studies of the global thermospheric response to solar and other influences. The temporal resolution of the data is 3–6 days. The estimated precision of the data is typically ±2%, but in isolated cases is as much as ±10%. The long-term accuracy of the data is estimated at ±5–10%, due to incomplete knowledge of drag coefficients and their variation with ambient composition. The data are available on the CEDAR Data System at http://cedarweb.hao.ucar.edu/.

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References


Figure 1. Examples of object selection based on mean motion behavior. Each panel shows the mean motion of an object as a function of time during the year 2006. The solid green curve shows the raw mean motion, and the dashed blue curve shows the mean motion with 60-day smoothing. The red square in the third panel indicates an outlier that was identified and eliminated. Each object’s Air Force catalog number is indicated in the panel, as are the monotonicity metric $M3$ (the percentage of TLE pairs with temporally increasing mean motion) and the drag-to-noise metric $DNR$ (the net change in mean motion during the year, divided by the variance around the smoothed mean motion). The red X’s indicate metrics with disqualifying values.

Figure 2. Examples of object selection based on ballistic coefficient stability. The plot shows time series of ballistic coefficients, estimated for each year based on a set of known reference objects, normalized to the overall average for each object. Examples are shown for eight objects; Air Force catalog numbers are given to the right of the plot. Objects whose time series are shown as red X’s were rejected by the algorithm. See text for details.
Figure 3. Average density ratios during 1989. (Top) The red curves show ratios in 2-day bins, for the height intervals listed to the right of the plots. The average number of objects that contributed to each bin is given below each height interval. The thickness of the green curves indicates the estimated uncertainty of the mean ratios. (Bottom) Daily $F_{10.7}$ (black) and $Ap$ (red) indices.

Figure 4. Average density ratios as a function of height during 1989. The red circles show averages in the same height bins used for Figure 3, for each of the indicated 10-day intervals. Error bars denote the estimated uncertainty of the mean. The blue curves show fits of averages to cubic splines with 150 km node spacing. The fits were constrained to have zero curvature at the upper (600 km) and lower (150 km) bounds.
Figure 5. Average density ratios during 1976. The red curves show averages in 2-day bins and the same 8 height intervals used for Figure 3; the thickness of the curves indicates the estimated uncertainty of the mean. The blue curves show the corresponding height-time fit described in section 3.1. The left panel shows results using the raw density ratios; the right panel shows results after implementing the quality control procedures described in section 3.2.
Figure 6. Distribution of density ratio residuals ($\alpha = \rho / \rho_{\text{MSIS}}$) for the selected objects and years indicated to the right of the plots. (Left panels) Histograms of the residual distributions (blue), and the Gaussian distribution (red) corresponding to the mean and standard deviation of the residuals; the red numbered tick marks indicate the number of standard deviations away from the mean. The mean and standard deviation of the residuals are given at the top of each panel, along with the normalized mean and standard deviation (see text for details); red X’s indicate disqualified objects. (Right panels) Quantile-normal plots of the residuals (blue dots). The position along the x-axis indicates the Gaussian-expected value of the residual, based on the residual’s ordered position in the set. The position along the y-axis indicates the actual value of the residual. The solid red line shows where residuals from a perfectly Gaussian distribution would fall. The linear correlation of the blue points is given in the upper left corner of each panel; red X’s indicate disqualified objects.
Figure 7. Variance of density ratio residuals for the selected years indicated to the right of the plots. The blue symbols show \(\sigma_j\), the standard deviation of residuals for individual objects. The red circles show the binned standard deviations, and the solid red lines show the interpolated reference standard deviation \(\sigma_R(z)\). The dotted red lines show the 2\(\sigma\) selection limit. See text for details.
Figure 8. Correlation of quantile-normal plots as a function of sample size; the x-axes are logarithmic scales. The top panels show the correlation, $r$. The bottom panels show $\ln(1-r)$. The left panels show results from a Monte Carlo simulation; for each simulated sample size, 100 trials were performed. The circles indicate the average values of $\ln(1-r)$ for each sample size, the solid red line shows a linear fit to $\ln(1-r)$, and the dashed red line shows the upper bound of the $2\sigma$ prediction interval. In the top panels, the averages and fit have been transformed back to the $r$ scale. The right panels show actual values computed from the residual density ratio distributions of 747 objects in 1996. The red lines are the same as in the left panels. Objects with $r$ values below the dotted red line were excluded from further processing.
Figure 9. Examples of the effects of the quality control procedure. The black lines in each panel show density ratios from a random sample of 100 selected objects with perigee heights in the intervals given to the right of the plots; data are from the indicated years. The green line shows the ensemble average of all objects. The red lines show density ratios from the highlighted objects indicated to the right of the plots. The left panels show the raw results, and the right panels show the results after applying the quality control criteria.
Figure 10. Sample autocorrelation of density ratio residuals from randomly selected objects in the indicated years and height ranges.

Figure 11. Distribution of sample correlation matrix values. The correlation values were computed among different pairs of objects in the indicated years and height ranges. The plots show the correlation values as a function of their position in the ordered series of correlations. The blue curves show the distribution of actual sample correlations. The red curve shows the distribution expected to occur, given the sample sizes, if the true correlation matrix were equal to the identity matrix (i.e., no correlation among data from different objects).
Figure 12. Examples of the quantities $\hat{\sigma}_{\text{bin}}$ and $\hat{\sigma}_{\text{fit}}^2$ (left); $N_{\text{obj}}$ and $N_{\text{eff}}$ (center); and $\sigma_{\text{bin}}$ and $\sigma_{\text{fit}}^2$ (right) described in section 4, for the selected years indicated to the right of the plots. The $\sigma$ values are expressed as percentages (i.e., the root-variance of the density ratios times 100). The red curves show results in 3-day temporal bins and 150 km altitude bins centered on the heights indicated to the right of the plots. The blue curves show results fitted as described in section 4 (equations 16 and 17).
Figure 13. Examples of fitted density ratios (blue) and uncertainties (green) for the same years and heights shown in Figure 12. The thickness of the green curves indicates the estimated uncertainty of the mean, which was computed using equation (17).

Figure 14. Examples of relative sampling density, as a function of local time and latitude, of the orbits used to compute the global mean density ratio. Darker areas indicate heavier representation. Results are shown for the dates and heights indicated. See text for details.

Figure 15. Global average absolute densities during 1989 from NRLMSISE-00 (orange) and derived from orbit data using the procedure described in this paper (blue). Results are shown for 250, 400, and 550 km, as indicated. The bottom panel shows the daily $F_{10.7}$ (solid black line), 81-day average $F_{10.7}$ (dashed black line) and $Ap$ (red) indices.
Figure 16. (Top) Orbit-derived log-density data (green dots) as a function of 26–34 nm EUV irradiance, as represented by the NRLSSI empirical model. The density data used in the figure were restricted to geomagnetically quiet days (daily Kp index less than 3). The symbols show binned averages; error bars denote the standard deviation. The solid curve is a fit of the data using cubic B splines (see text for details). (Middle) Same as the top panel, but as a function of the \( F_{10.7} \) index. (Bottom) The root-mean-square of residuals around the EUV fit (red) and \( F_{10.7} \) fit (blue), computed in bins of \( F_{10.7} \).

Figure 17. Average temporal span \((t_j - t_i)\) covered by the drag integrations of equation (1), as a function of year and height.