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Tutorial on Plasma-Neutral Interactions in the MLT-X

[MLT-eXtended from the Upper Mesosphere through the Middle Thermosphere (80-200 km)]

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The Universality of Plasma-Neutral Interactions

- Planetary Space-Atmosphere Interaction Regions
- Stellar Chromospheres
- Dusty Plasmas
- Interplanetary Space Weather (Planetary Habitability)
- Interstellar Space Weather (Exoplanets)
Tutorial Outline

- Earth’s Space-Atmosphere Interaction Regions (SAIR)
  - Weakly Ionized Gas Description
  - Transport Equations
  - Ionosphere Plasma Motion
  - Conductivity and Currents
  - Frictional and Joule Heating Rates

- Our Sun’s Chromosphere
  - Weakly Ionized Gas Description
  - Plasma Mobilities
  - Joule Heating Rates
SAIR Plasma-Neutral Interactions

Plasma-Neutral Chemistry

Plasma-Neutral Drag Forces

Plasma-Neutral Frictional Heating

Plasma-Neutral Electrodynamics
Earth’s Ionosphere/Thermosphere

Partialy Ionized Gas Above 250 km – plasma-neutral and Coulomb interactions are equally important

Weakly Ionized Gas (or a strongly neutral gas) below 250 km – plasma-neutral interactions dominate over Coulomb interactions
Upper planetary atmospheres must include coupled equations for the neutral gas and plasma (electron and ion) typically defined in a rotating coordinate system.

The equations become system dependent when considering the collisional aspects of the environment and the definition of an average velocity by which all higher moments are defined.
Upper Atmosphere Transport System of Equations
(Schunk and Nagy: Ionospheres Physics, Plasma Physics, and Chemistry)

Taking velocity moments of the Boltzmann Equation leads to various forms of the transport equations:

- **5th moment** \((n=1, u=3, T=1)\): assumed drifting Maxwellian distribution with isotropic pressure and all higher order moments neglected

- **13th moment** \((n=1, u=3, T=1, q=3, \tau=5)\) assumed small departure from Maxwellian with higher order moments expressed in terms of the five variables

- **20th moment** \((n=1, u=3, T=1, \tau=5, Q=10)\) assumed large departure from Maxwellian with the full heat flux tensor required to adequately represent the gas or plasma behavior
Maxwell’s Equations

Maxwell Equations in vacuum

\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \quad (Gauss' \; s \; Law) \]
\[ \nabla \cdot \vec{B} = 0 \]
\[ \frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} \quad (Faraday'\; s \; Law) \]
\[ \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \nabla \times \vec{B} - \mu_0 \vec{J} \quad (Ampere'\; s \; Law) \]

Conservation of Charge

\[ \nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t} \approx 0 \]

Maxwell Equations applied to Ionosphere

\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \]
\[ \nabla \cdot \vec{B} = 0 \]
\[ \nabla \times \vec{E} = 0 \quad (\vec{E} = -\nabla \Phi) \]
\[ \nabla \times \vec{B} = \mu_0 \vec{J} \]

Ionosphere Ohm’s Law (in neutral frame)

\[ \vec{j} = \sigma_P \vec{E}_\perp - \sigma_H \frac{\vec{E}_\perp \times \vec{B}}{B} + \sigma_{||} \vec{E}' \]

Charge Neutrality

\[ n_e \approx \sum_{\text{ions}} n_i \]
Upper Atmosphere Transport System of Equations

Two main Points to Remember Throughout this Tutorial:

Define your coordinate system and reference frame:
- Determined for physical, mathematical, or numerical convenience
  - Pressure coordinates in the vertical
  - Earth-fixed frame
  - Rotating reference frame

Understand your coordinate system and reference frame:
- Physical Interpretation
  - Earth-fixed frame is non-inertial: requires Coriolis and Centripetal accelerations
  - Average velocity definition of the system (important for defining higher order velocity moments (pressure, temperature, viscosity) and describing behavior in a multiconstituent gas
Multiconstituent Momentum Eqs for a Weakly Ionized Gas

**Neutrals:**

\[
n_n m_n \frac{d\vec{U}_n}{dt} = -\nabla \rho_n \nabla \cdot \vec{\tau}_n + n_n m_n \left[ \vec{g} - 2\vec{\Omega} \times \vec{U}_n - \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) \right] - n_n m_n \nu_{ni} (\vec{U}_n - \vec{V}_i)
\]

**Ions:**

\[
n_i m_i \frac{d\vec{V}_i}{dt} = -\nabla \rho_i \nabla \cdot \vec{\tau}_i + n_i m_i \vec{g} + e n_i (\vec{E} + \vec{V}_i \times \vec{B}) - n_i m_i \nu_{in} (\vec{V}_i - \vec{U}_n)
\]

**Electrons:**

\[
n_e m_e \frac{d\vec{V}_e}{dt} = -\nabla \rho_e \nabla \cdot \vec{\tau}_e + n_e m_e \vec{g} + e n_e (\vec{E} + \vec{V}_e \times \vec{B}) - n_e m_e \nu_{en} (\vec{V}_e - \vec{U}_n)
\]
I/T: Plasma-Neutral Collisions

\[ k = \frac{\Omega}{\nu}, \text{mobility} \quad \Omega = \frac{qB}{m}, \text{gyrofrequency} \]

\[ \nu_{en}, \text{electron-neutral collision frequency} \quad \nu_{in}, \text{ion-neutral collision frequency} \]

Ion and Electron Momentum Eqs for a Weakly Ionized Gas

Ion and electron momentum equation accounts for pressure gradient, gravitational, electric, magnetic, and collisional forces

\[
\begin{align*}
    n_i m_i \frac{d\vec{V}_i}{dt} &= -\nabla \bar{P}_i + n_i m_i \bar{g} + e n_i \left( \vec{E} + \vec{V}_i \times \vec{B} \right) - n_i m_i \nu_{in} \left( \vec{V}_i - \bar{U}_n \right) \\
    n_e m_e \frac{d\vec{V}_e}{dt} &= -\nabla \bar{P}_e + n_e m_e \bar{g} + e n_e \left( \vec{E} + \vec{V}_e \times \vec{B} \right) - n_e m_e \nu_{en} \left( \vec{V}_e - \bar{U}_n \right)
\end{align*}
\]

Assume only static E- and B-fields and neutral collisions with \( U_n = 0 \),

Ions:

\[
\vec{V}_i = \frac{1}{1 + k_i^2} \left\{ k_i \frac{\vec{E}}{B} + \left( \frac{k_i}{B} \right)^2 \vec{E} \times \vec{B} + \left( \frac{k_i}{B} \right)^3 \left( \vec{E} \cdot \vec{B} \right) \vec{B} \right\} \quad \text{where} \quad k_i = \frac{\Omega_i}{\nu_{in}}
\]

Electrons:

\[
\vec{V}_e = \frac{1}{1 + k_e^2} \left\{ -k_e \frac{\vec{E}}{B} + \left( \frac{k_e}{B} \right)^2 \vec{E} \times \vec{B} - \left( \frac{k_e}{B} \right)^3 \left( \vec{E} \cdot \vec{B} \right) \vec{B} \right\} \quad \text{where} \quad k_e = \frac{\Omega_e}{\nu_{en}}
\]
Plasma Drift Measurements – “Frozen-In” Flux

\[
V_{e,i}^\perp = \frac{\vec{E} \times \vec{B}}{B^2}
\]

when \( k_i, k_e >> 1 \).
Ionosphere Plasma Motion and Currents

\[ \vec{V}_i \perp = \frac{1}{1 + k_i^2} \left\{ k_i \vec{E} + \left( \frac{k_i}{B} \right)^2 \vec{E} \times \vec{B} \right\} \]

- Ion motion perpendicular to B rotates towards the electric field as collision frequency increases with decreasing altitude.

- Ion magnitude perpendicular to B decreases with increasing collision frequency (to a point).

- Currents perpendicular to B rotate towards the \(-ExB\) direction.

- Current magnitude increases with increasing collision frequency (to a point).
Observed E-region Ion Motion and Electron Density

Where the Frozen-In Flux gets Slushy

\[ \vec{v}_{i+}^{\perp} = \frac{1}{1 + k_i^2} \left\{ \frac{k_i}{B} \vec{E} + \left( \frac{k_i}{B} \right)^2 \vec{E} \times \vec{B} \right\} \]
Observed E-region Ion Motion and Electron Density

Where the Frozen-In Flux gets Slushy

\[ \vec{V}_{i}^{\perp} = \frac{1}{1 + k_{i}^{2}} \left( \frac{k_{i}}{B} \vec{E} + \left( \frac{k_{i}}{B} \right)^{2} \vec{E} \times \vec{B} \right) \]
Currents and Pedersen Conductivity Resolved in the E-region Ionosphere

\[ j_\perp(z) = eN_e(z)(\vec{V}_i\perp(z) - \vec{V}_e\perp) \]
Currents and Pedersen Conductivity Resolved in the E-region Ionosphere

\[ j_\perp (z) = eN_e (z) (\vec{V}_i \perp (z) - \vec{V}_e \perp) \]
Currents and Neutral Winds

Thayer, J.P., High latitude currents and their energy exchange with the ionosphere-thermosphere system, JGR 2000

Thayer, J.P., Height-resolved Joule heating rates in the high-latitude E region and the influence of neutral winds, JGR 1998
Electrodynamics with Neutral Winds

Ion momentum equation:
\[ \vec{V}_i' = \vec{V}_i - \vec{U}_n = \frac{1}{1 + k_i^2} \left\{ k_i \vec{E}' + \frac{k_i^2}{B} \vec{E}' \times \vec{B} + \frac{k_i^3}{B^3} (\vec{E}' \cdot \vec{B}) \vec{B} \right\} \]

Electron momentum equation:
\[ \vec{V}_e' = \vec{V}_e - \vec{U}_n = \frac{1}{1 + k_e^2} \left\{ -k_e \vec{E}' + \frac{k_e^2}{B} \vec{E}' \times \vec{B} - \frac{k_e^3}{B^3} (\vec{E}' \cdot \vec{B}) \vec{B} \right\} \]

Current density:
\[ \vec{j} = en_e (\vec{V}_i - \vec{V}_e) = en_e (\vec{V}_i - \vec{U}_n - (\vec{V}_e - \vec{U}_n)) = en_e (\vec{V}_i' - \vec{V}_e') = \vec{j}' \]
\[ \vec{j} = en_e \left\{ \left( \frac{k_e}{1 + k_e^2} + \frac{k_i}{1 + k_i^2} \right) \vec{E}' - \left( \frac{k_e}{1 + k_e^2} - \frac{k_i}{1 + k_i^2} \right) \frac{\vec{E}' \times \vec{B}}{B^2} + \left( \frac{k_e}{1 + k_e^2} + \frac{k_i}{1 + k_i^2} \right) \frac{(\vec{E}' \cdot \vec{B}) \vec{B}}{B^3} \right\} \]

Ionospheric Ohm’s Law (in neutral frame):
\[ \vec{j} = \sigma_P \vec{E}_\perp - \sigma_H \frac{\vec{E}_\perp \times \vec{B}}{B} + \sigma_{||} \vec{E}' \quad \text{where} \quad \vec{E}' = \vec{E} + \vec{U}_n \times \vec{B} \]
Interpretation of Ohm’s Law is Reference Frame Dependent

Generalized Ohm’s Law:
\[ \frac{\partial \vec{j}}{\partial t} = \text{pressure term} + \text{gravity term} + \text{electric field term} + \text{current density term} + \ldots \]

Ionospheric Ohm’s Law:
\[ \vec{j} = \sigma_p \vec{E}_\perp - \sigma_H \frac{\vec{E}_\perp \times \vec{B}}{B} + \sigma_\parallel \vec{E}_\parallel \]

Ohm’s Law (in neutral frame): \[ \vec{E}' = \vec{E} + \vec{U}_n \times \vec{B} \]

Ohm’s Law (in plasma frame): \[ \vec{E}^* = \vec{E} + \vec{V}_p \times \vec{B} \]

\( \sigma_{\text{neutral}} \neq \sigma_{\text{plasma}} \), and \( \vec{E}' \neq \vec{E}^* \)

(Song et al., JGR 2001)
Pedersen Mobility (in Neutral Frame)

$$\mu_p = \sigma_p \frac{B}{eN_e} = \left( \frac{k_i}{1 + k_i^2} + \frac{k_e}{1 + k_e^2} \right)$$

$$k_i = \frac{\Omega_i}{v_{in}} \quad k_e = \frac{\Omega_e}{v_{en}}$$

$$\mu_p = \left( \frac{k_i}{1 + k_i^2} + \frac{k_e}{1 + k_e^2} \right)$$
- Hall conductivity peaks in lower ionosphere below 120 km
  - Essentially removed at night (unless aurora)
- Pedersen conductivity distributed in two regions
  - E-region greater than F-region during the daytime
  - F region greater than E region at night.
- Parallel conductivity greater than transverse conductivities everywhere above 90 km.
Neutral Frictional Heating

Neutral Energy Eq

$$\frac{\delta E_n}{\delta t} = \sum_{i} n_n m_n \nu_{ni} \left[ 3k_B (T_i - T_n) + m_i \left( \vec{V}_i - \vec{U}_n \right)^2 \right] + \ldots$$

$$\frac{\delta E_n}{\delta t} = \sum_{i} n_n m_n \nu_{ni} \left[ m_n \left( \vec{V}_i - \vec{U}_n \right)^2 + m_i \left( \vec{V}_i - \vec{U}_n \right)^2 \right] + \ldots$$

Ion Energy Eq

$$\frac{\delta E_i}{\delta t} = 0$$

$$\therefore 3k_B (T_i - T_n) \approx m_n \left( \vec{V}_i - \vec{U}_n \right)^2$$

$$n_n m_n \nu_{ni} \approx n_i m_i \nu_{in} \quad \text{and} \quad m_n \approx m_i$$

Approximate Neutral Frictional Heating

$$\frac{\delta E_n}{\delta t} = \sum_{i} n_i m_i \nu_{in} \left( \vec{V}_i - \vec{U}_n \right)^2 \quad \text{[W/m}^3\text{]}$$

(Thayer and Semeter, JASTP 2004)
Joule Heating Rate (Neutral Frame)

\[ \mathbf{j}' \cdot \mathbf{E}' = \sum_i en_i (\mathbf{V}'_i - \mathbf{V}_e') \cdot \mathbf{E}' \]

Assume \( k_e \) is large, valid above 80 km:

\[ \mathbf{j}' \cdot \mathbf{E}' = \sum_i en_i \left( \frac{k_e}{1 + k_e^2} + \frac{k_i}{1 + k_i^2} \right) \frac{\mathbf{E}'}{B} \cdot \mathbf{E}' = \sigma_p \mathbf{E}'^2 \]

Recall, the magnitude of the ion velocity from the momentum equation is

\[ V_i'^2 = (\mathbf{V}_i' - \mathbf{U}_n)^2 = \frac{k_i^2}{1 + k_i^2} \frac{E_i'^2}{B^2} \]

Solve for \( \frac{E_i'^2}{B^2} \),

\[ \mathbf{j}' \cdot \mathbf{E}' = \sum_i m_i n_i \Omega_i \left( \frac{k_i}{1 + k_i^2} \right) \frac{E_i'^2}{B^2} \]

Joule Heating Rate is “Equal” to the Neutral Frictional Heating rate

\[ \mathbf{j}' \cdot \mathbf{E}' = \sum_i n_i m_i \nu_{in} (\mathbf{V}_i' - \mathbf{U}_n)^2 \]

\[ \frac{\delta E_n}{\delta t} = \sum_i n_i m_i \nu_{in} (\mathbf{V}_i' - \mathbf{U}_n)^2 \]
Interpretation of Joule Heating is Reference Frame Dependent


Uses plasma velocity as their rest frame

\[ \dot{q}_j = j \cdot (E + \vec{V}_e \times \vec{B}) + \sum_i m_i n_i v_{in} (\vec{V}_i - \vec{U}_n)^2 = \eta J^2 + \sum_i m_i n_i v_{in} (\vec{V}_i - \vec{U}_n)^2 \]

Note:

Resistivity: \( \eta = \frac{m_e v_e}{e^2 n_e} = \frac{B}{en_e \Omega_e} \), \( \ll 1 \) above 90km

Therefore, their form agrees with the form we just derived but approached differently,

\[ \dot{q}_j = \sum_i m_i n_i v_{in} (\vec{V}_i - \vec{U}_n)^2 \]
Plasma-Neutral Interactions: Solar Chromosphere

- **Temperature (K)**: The temperature decreases from the photosphere to the chromosphere and then sharply increases in the transition region.
- **Density**: The density decreases from the photosphere to the chromosphere and then remains relatively constant in the transition region.
- **Mass Density (kg m⁻³)**: The mass density decreases from the photosphere to the chromosphere and then remains relatively constant in the transition region.

**Graphs**:
- **Wavelength (nm) vs. W/m²/nm**: Shows the variation of energy flux with wavelength, indicating the transitions between different regions of the solar atmosphere.
Chromosphere / Ionosphere Comparison

Weakly Ionized Gas

Solar Chromosphere

Earth’s Ionosphere / Thermosphere
Plasma – Neutral Collisions

Solar Chromosphere

Earth’s Ionosphere / Thermosphere

Normalized Pressure

Frequency (Hz)

Normalized Pressure

Frequency (Hz)
Charge Mobilities

Solar Chromosphere

Earth’s Ionosphere / Thermosphere

\[ \mu_P = \left( \frac{k_i}{1 + k_i^2} + \frac{k_e}{1 + k_e^2} \right) \]

\[ \mu_H = \left( \frac{k_e^2}{1 + k_e^2} - \frac{k_i^2}{1 + k_i^2} \right) \]
Solar Chromosphere: Joule Heating

\[ \frac{j^2}{\sigma_c}, \text{where} \ \sigma_c = \sigma_p + \frac{\sigma_H}{\sigma_p} \]

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Summary

Define and know your coordinate system and reference frame in order to properly interpret plasma-neutral interactions, enabling more universal interpretation of:

- Planetary Ionospheres / Thermospheres
- Stellar Chromospheres

Ionospheric Ohm’s Law provides no causal relationship but simply states the current and electric field are linearly related by conductivities defined for the given reference frame.

Frictional heating may be a more appropriate description of energy transfer in the multiconstituent Ionosphere/Thermosphere system than Joule heating.

- Can have neutral, ion, and electron frictional heating. However, increased plasma temperatures will transfer heat to the neutral gas due to temperature differentials. With some approximations this overall heat transfer to the neutrals can be equated to Joule heating (but not in the classical sense).

Leake et al. (2014), Ionized Plasma and Neutral Gas Coupling in the Sun’s Chromosphere and Earth’s Ionosphere/Thermosphere, Space Science Reviews.
Plasma-Neutral Interaction Challenges in the 80 - 200km Domain

- “Missing” energy in M-I coupling (lacking adequate conductivity descriptions with dependencies on electron and neutral density)
- “Transforming” energy in I-T coupling (lacking sufficient neutral wind observations to determine energy dissipation and generation)
- “Modifying” dynamo processes (lacking neutral wind observations coincident with plasma measurements)
- “Profiling” neutral and plasma properties in near-spatial and temporal simultaneity (lacking vertical structure description, i.e. gradients)