Geospace Electrodynamics

Roger H. Varney

SRI International

June 20, 2016
Maxwell’s Equations

\[ \nabla \cdot \mathbf{E} = \frac{\rho_c}{\varepsilon_0} \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
\[ \nabla \cdot \mathbf{B} = 0 \]
\[ \nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J} \]
\[ \frac{\partial \rho_c}{\partial t} + \nabla \cdot \mathbf{J} = 0 \]

- Solutions where \( \rho_c = 0 \) and \( \mathbf{J} = 0 \) are a completely solved problem
- Solutions where \( \rho_c \) and \( \mathbf{J} \) are known a priori are a completely solved problem
- In media (like geospace plasmas) \( \mathbf{J} \) depends on the fields \( \mathbf{E} \) and \( \mathbf{B} \)
- A generalized Ohm’s law (GOL) relating \( \mathbf{J} \) to \( \mathbf{E} \) and \( \mathbf{B} \) is needed to close the system of equations
Vlasov - Maxwell Equations

\[
\begin{align*}
\frac{\partial f_e}{\partial t} + v \cdot \nabla f_e + \left[-\frac{e}{m_e} (E + v \times B) + g\right] \cdot \nabla_v f_e &= \frac{\delta f_e}{\delta t} \\
\frac{\partial f_i}{\partial t} + v \cdot \nabla f_i + \left[\frac{q_i}{m_i} (E + v \times B) + g\right] \cdot \nabla_v f_i &= \frac{\delta f_i}{\delta t}
\end{align*}
\]

\[J = \sum_i q_i \int v f_i \, dv - e \int v f_e \, dv\]

\[\nabla \times E = -\frac{\partial B}{\partial t}\]

\[\nabla \times B = \frac{1}{c^2} \frac{\partial E}{\partial t} + \mu_0 J\]

- \(f (x, v, t)\) are 7-dimensional particle distribution functions
- \(\frac{\delta}{\delta t}\) denotes collisional terms
- Completely impractical to use in most situations
Theories of geospace electrodynamics differ depending on:

- Inclusion of displacement current $\frac{\partial \mathbf{E}}{\partial t}$
  - Only important for radio-waves and high-frequency phenomena

- Inclusion of inductive fields $\frac{\partial \mathbf{B}}{\partial t}$
  - Electrostatic approximation common in ionosphere

- Approximations of particle motion (simplifications of the GOL)
  - Fluid vs kinetic
  - Guiding center approximation
  - Adiabatic assumptions
Areas of Geospace Electrodynamics

1. Ionospheric Electrostatics
2. Inner Magnetospheric Kinetic Electrodynamics
3. Magnetohydrodynamics
4. Solar Wind-Magnetosphere-Ionosphere Coupling
5. The Polar Wind and Auroral Acceleration Region
Electrostatic Approximation

\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \rightarrow \nabla \cdot \mathbf{J} = 0 \quad \text{(Recall: } \nabla \cdot \nabla \times \mathbf{B} = 0) \]

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \rightarrow \mathbf{E} = -\nabla \Phi \quad \text{(Recall: } \nabla \times \nabla \Phi = 0) \]

Ohm's Law for the ionosphere:

\[ \mathbf{J} = \sigma \cdot \mathbf{E} + \mathbf{J}_0 \]

Putting everything together yields a boundary value problem:

\[ \nabla \cdot \sigma \cdot \nabla \Phi = \nabla \cdot \mathbf{J}_0 \]
Motion of Particles in Uniform Fields: $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

**Uniform B Field**

- Electrons
- Ions

\[ \mathbf{B} = B_z \hat{z} \]

**Crossed Uniform E and B**

- Electric field ($\mathbf{E}$) pointing up
- Magnetic field ($\mathbf{B}$) pointing along $\hat{z}$

\[ \mathbf{v}_D = \frac{\mathbf{E} \times \mathbf{B}}{B^2} \]

Note $\mathbf{E} + \frac{\mathbf{E} \times \mathbf{B}}{B^2} \times \mathbf{B} = 0$ as long as $\mathbf{E} \cdot \mathbf{B} = 0$
Effects of Collisions: Ohm’s Law for the Ionosphere

Steady-state momentum equation for each species (zero neutral wind case):

\[ 0 = n_\alpha q_\alpha (E + u_\alpha \times B) - \nu_{\alpha n} m_\alpha n_\alpha u_\alpha \]

Resulting Ohm’s Law:

\[ J = \sum_\alpha n_\alpha q_\alpha u_\alpha \rightarrow J = \begin{pmatrix} \sigma_P & -\sigma_H & 0 \\ \sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_0 \end{pmatrix} \cdot E \]
Other Kinds of Current: Complete Dynamo Equation

Substitute \( \mathbf{F} \) for \( q_\alpha \mathbf{E} \) in steady state momentum equation.

- Wind drag: \( \mathbf{F} = \nu_\alpha n_\alpha \mathbf{u}_n \rightarrow \mathbf{J} = \sigma \cdot (\mathbf{u}_n \times \mathbf{B}) \)
- Gravity: \( \mathbf{F} = m_\alpha \mathbf{g} \rightarrow \mathbf{J} = \Gamma \cdot \mathbf{g} \)

Pressure Gradients (Diamagnetic Currents):

\[
\mathbf{F} = -\frac{1}{n_\alpha} \nabla p_\alpha \rightarrow \mathbf{J} = \mathbf{D} \cdot \nabla \sum_\alpha p_\alpha
\]

Complete Dynamo Equation:

\[
\nabla \cdot \sigma \cdot \nabla \Phi = \nabla \cdot \left( \sigma \cdot (\mathbf{u}_n \times \mathbf{B}) + \Gamma \cdot \mathbf{g} + \mathbf{D} \cdot \nabla \sum_\alpha p_\alpha \right)
\]
Slab Models of F- and E-region Dynamos

**F-region**

\[ J = \sigma_P (E + u_n \times B) \]

\[ J = 0 \rightarrow E = -u_n \times B \]

\[ v_D = \frac{E \times B}{B^2} = \frac{-u_n \times B \times B}{B^2} = u_n \]

**E-region**

A vertical electric field forms to oppose the vertical Hall current.

\[ \sigma_H E_x = \sigma_P E_z \rightarrow E_z = \frac{\sigma_H}{\sigma_P} E_x \]

The Hall current from this new \( E_z \) adds to the existing Pedersen current from \( E_x \)

\[ J_x = \sigma_H E_z + \sigma_P E_x \]

\[ = \left( (\sigma_H/\sigma_P)^2 + 1 \right) \sigma_P E_x \equiv \sigma_C E_x \]
Closure of Field Aligned Currents in a Slab Ionosphere

3D potential equation with magnetospheric currents:

\[ \nabla \cdot \sigma \cdot \nabla \Phi = \nabla \cdot J^{\text{iono}} + \nabla \cdot J^{\text{mag}} \]

Integrate over altitude, assume equipotential field lines:

\[ \nabla_\perp \cdot \Sigma \cdot \nabla_\perp \Phi = \int \nabla \cdot J^{\text{iono}} \, dz + \int \nabla \cdot J^{\text{mag}} \, dz \]

\[ \text{K}^{\text{iono}} \equiv \int J^{\text{iono}} \, dz \]

Expand the divergence:

\[ \nabla \cdot J^{\text{mag}} = \nabla_\perp \cdot J^{\text{mag}}_\perp + \frac{\partial J^{\text{mag}}_\parallel}{\partial z} \]

Above ionosphere, \( J^{\text{mag}}_\perp = 0 \)

\[ \int \nabla \cdot J^{\text{mag}} \, dz = J^{\text{mag}}_\parallel \]

2D slab ionosphere potential equation:

\[ \nabla_\perp \cdot \Sigma \cdot \nabla_\perp \Phi = \nabla_\perp \cdot \text{K}^{\text{iono}} + J^{\text{mag}}_\parallel \]
Conjugacy and Mapping

In low latitudes current out of northern hemisphere (N) equals current into southern hemisphere (S)

\[ J_N^\parallel = -J_S^\parallel \]

Assuming equipotential field lines:

\[
\nabla \perp \cdot \Sigma^N \cdot \nabla \perp \Phi - \nabla \cdot K^{N\text{iono}} \\
\n= -\nabla \perp \cdot \Sigma^S \cdot \nabla \perp \Phi + \nabla \cdot K^{S\text{iono}} \\
\n\nabla \perp \cdot (\Sigma^N + \Sigma^S) \cdot \nabla \perp \Phi \\
\n= \nabla \cdot (K^{N\text{iono}} + K^{S\text{iono}})
\]

Otsuka et al. (2004)
Equatorial Fountain Effect
Influences of Atmospheric Tides (Immel et al. 2006)
Magnetic Mirror Force and Bounce Motion

\[ \mathbf{F} = -\frac{m v^2}{2B} \nabla B \]
Gradient-Curvature Drift

Gradient Drift
\[ \mathbf{v}_{\nabla B} = -\frac{m\mathbf{v}_\parallel^2}{2qB^3} \nabla B \times \mathbf{B} \]

Curvature Drift
\[ \mathbf{v}_c = -\frac{m\mathbf{v}_\parallel^2}{qB^2} \left[ \frac{\mathbf{B}}{B} \cdot \nabla \left( \frac{\mathbf{B}}{B} \right) \right] \times \mathbf{B} \]

- Both drifts are energy dependent
- Both drifts move ions CW and electrons CCW
Adiabatic Invariants

<table>
<thead>
<tr>
<th>Type of Periodic Motion</th>
<th>Adiabatic Invariant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gyromotion</td>
<td>$\mu = \frac{mv_\perp^2}{2B}$</td>
</tr>
<tr>
<td>Bounce Motion</td>
<td>$J = \oint_{\text{Bounce}} mv_\parallel ds$</td>
</tr>
<tr>
<td>Drift Motion</td>
<td>$\Phi = \oint_{\text{Drift}} qA \cdot ds$</td>
</tr>
</tbody>
</table>

Average over periodic motion to reduce the dimensionality of the problem

Velocity-like coordinates:

<table>
<thead>
<tr>
<th>Energy</th>
<th>$\mu$</th>
<th>gyrophase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Avg. over gyromotion</td>
</tr>
</tbody>
</table>

Position coordinates:

<table>
<thead>
<tr>
<th>L-shell</th>
<th>pos. along field line</th>
<th>MLT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Avg. over bounce motion</td>
<td>Avg. over drift motion</td>
</tr>
</tbody>
</table>
Breaking the Adiabatic Invariants: Wave Environment

Cumulative effect of wave particle interactions modeled as phase-space diffusion coefficients

Images courtesy the U. of Iowa EMFISIS Team
Collective behavior in the inner magnetosphere

- Gradient-curvature drifts result in currents
- Currents affect fields via
  \[ \mathbf{J} = \frac{1}{\mu_0} \nabla \times \mathbf{B} \]
- Fields affect particle gradient-curvature drifts
\( \nabla \cdot \mathbf{J} = 0 \) still applies

\[
J_\parallel = \nabla_\perp \cdot \int J_\perp \, ds
\]

This current closes in both ionospheres

\[
\zeta J_\parallel = \nabla_\perp \cdot \Sigma^N \cdot \nabla_\perp \Phi - \nabla_\perp \cdot K^{N_{iono}}
\]

\[
(1 - \zeta) J_\parallel = \nabla_\perp \cdot \Sigma^S \cdot \nabla_\perp \Phi - \nabla_\perp \cdot K^{S_{iono}}
\]

\[
J_\parallel = \nabla_\perp \cdot (\Sigma^N + \Sigma^S) \cdot \nabla_\perp \Phi
\]

\[
- \nabla_\perp \cdot (K^{N_{iono}} + K^{S_{iono}})
\]

Solve boundary-value problem for \( \Phi \) to get \( \mathbf{E} \)-fields in ionosphere and inner-magnetosphere.
2-Fluid Equations

Describe ions and electrons as separate fluids:

\[
\frac{\partial}{\partial t} n_e + \nabla \cdot [n_e u_e] = 0
\]

\[
\frac{\partial}{\partial t} (m_e n_e u_e) + \nabla \cdot [m_e n_e u_e + p_e I] = -en_e (E + u_e \times B) + R_{e}^{\text{coll}}
\]

\[
\frac{\partial}{\partial t} \left( \frac{1}{2} m_e n_e u_e^2 + \frac{3}{2} p_e \right) + \nabla \cdot \left[ \frac{1}{2} m_e n_e u_e^2 u_e + \frac{5}{2} p_e u_e \right] = -m_e n_e e u_e \cdot \left[ E - \frac{R_{e}^{\text{coll}}}{en_e} \right]
\]

\[
\frac{\partial}{\partial t} n_i + \nabla \cdot [n_i u_i] = 0
\]

\[
\frac{\partial}{\partial t} (m_i n_i u_i) + \nabla \cdot [m_i n_i u_i + p_i I] = en_i (E + u_i \times B) + R_{i}^{\text{coll}}
\]

\[
\frac{\partial}{\partial t} \left( \frac{1}{2} m_i n_i u_i^2 + \frac{3}{2} p_i \right) + \nabla \cdot \left[ \frac{1}{2} m_i n_i u_i^2 u_i + \frac{5}{2} p_i u_i \right] = m_i n_i e u_i \cdot \left[ E + \frac{R_{i}^{\text{coll}}}{en_i} \right]
\]
Plasma as A Single Fluid

Define single fluid quantities:

\[ \rho = m_i n_i + m_e n_e \]
\[ \mathbf{u} = \frac{m_i n_i \mathbf{u}_i + m_e n_e \mathbf{u}_e}{m_i n_i + m_e n_e} \]
\[ p = p_i + p_e \]
\[ \mathbf{J} = e n_i \mathbf{u}_i - e n_e \mathbf{u}_e \]

Make a few approximations

Quasineutrality:
\[ n_e = n_i \]
Mass Ratio:
\[ \frac{m_e}{m_i} \ll 1 \]

\[ \rightarrow \rho \approx m_i n_i \]
\[ \rightarrow \mathbf{u} \approx \mathbf{u}_i \]
\[ \rightarrow \mathbf{J} \approx e n (\mathbf{u} - \mathbf{u}_e) \]

With these definitions and approximations the 2-fluid equations can be rearranged into the Extended MHD equations.
Extended MHD

\[
\begin{align*}
\frac{\partial}{\partial t} \rho &+ \nabla \cdot [\rho \mathbf{u}] = 0 \\
\frac{\partial}{\partial t} \rho \mathbf{u} &+ \nabla \cdot [\rho \mathbf{uu} + \rho \mathbf{I}] = \mathbf{J} \times \mathbf{B} \\
\frac{\partial}{\partial t} \left( \frac{p}{\rho^{2/3}} \right) &+ \nabla \cdot \left[ \mathbf{u} \frac{p}{\rho^{2/3}} \right] = \frac{2}{3} \rho^{-2/3} \\
\frac{\partial}{\partial t} \mathbf{B} &+ \nabla \times \mathbf{E} = 0 \\
\frac{\partial}{\partial t} \mathbf{E} &- c^2 \nabla \times \mathbf{B} = -\frac{1}{\epsilon_0} \mathbf{J}
\end{align*}
\]

- All \( \frac{\partial}{\partial t} \) terms retained in derivation
- This set of equations can be used for initial value problems
Limiting Cases of the GOL

\[
\frac{m_e}{e^2 n} \left\{ \frac{\partial}{\partial t} \mathbf{J} + \nabla \cdot \left[ \mathbf{J} \mathbf{u} + \mathbf{u} \mathbf{J} - \frac{1}{en} \mathbf{J}_e \mathbf{J} \right] \right\} = \mathbf{E} + \mathbf{u} \times \mathbf{B} + \frac{1}{en} \nabla p_e - \frac{1}{en} \mathbf{J} \times \mathbf{B} - \frac{m_e \nu e_i}{e^2 n} \mathbf{J}
\]

- **Electron Inertia**: negligible on length scales \( \lambda_e = \sqrt{\frac{m_e}{e^2 n \mu_0}} \)
- **Ambipolar Field**: negligible in cold plasma
- **Hall Term**: negligible on length scales \( \lambda_i = \sqrt{\frac{m_i}{e^2 n \mu_0}} \) in collisionless plasma
- **Resistive Term**: negligible in collisionless plasma
Ideal MHD

Assumptions:
- \( \mathbf{E} + \mathbf{u} \times \mathbf{B} = 0 \)
- \( \mathbf{J} = \frac{1}{\mu_0} \nabla \times \mathbf{B} \)

Equations:

\[
\begin{align*}
\frac{\partial}{\partial t} \rho + \nabla \cdot [\rho \mathbf{u}] &= 0 \\
\frac{\partial}{\partial t} \rho \mathbf{u} + \nabla \cdot [\rho \mathbf{uu} + p \mathbf{l}] &= \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} \\
\frac{\partial}{\partial t} \left( \frac{p}{\rho^{2/3}} \right) + \nabla \cdot \left[ \mathbf{u} \frac{p}{\rho^{2/3}} \right] &= \frac{2}{3} \rho^{-2/3} \\
\frac{\partial}{\partial t} \mathbf{B} - \nabla \times [\mathbf{u} \times \mathbf{B}] &= 0
\end{align*}
\]
Magnetic Tension, Magnetic Pressure, and Alfvén Waves

Shear Alfvén Waves

\[ v_A = \frac{B}{\sqrt{\mu_0 \rho}} \]

Compressional Alfvén Waves (Magnetosonic Waves)

\[ v_M = \sqrt{v_s^2 + v_A^2} \]
Flux Tubes and the Frozen-in Condition

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) \implies \frac{D\phi_m}{Dt} = 0
\]

In electrostatic fields

\[
\mathbf{u} = \frac{1}{B^2} \left( -\nabla \Phi \times \mathbf{B} \right)
\]

\[
\nabla \times (\mathbf{u} \times \mathbf{B}) = 0
\]

The flux tubes expand and contract to always enclose the same flux

In inductive fields

\[
\nabla \times (\mathbf{u} \times \mathbf{B}) \neq 0
\]

The magnetic field changes to preserve the enclosed flux
\[ \frac{m_e}{e^2 n} \left\{ \frac{\partial}{\partial t} \mathbf{J} + \nabla \cdot \left[ \mathbf{J} u + u \mathbf{J} - \frac{1}{en} \mathbf{J} \mathbf{J} - \frac{e}{m_e} \mathbf{P}_e \right] \right\} = \mathbf{E} + u \times \mathbf{B} - \frac{1}{en} \mathbf{J} \times \mathbf{B} \]
MHD diamagnetic currents are a poor approximation of the ring current because using a single MHD pressure misses the energy and pitch-angle dependences of the gradient-curvature drift.
Recovering the Ionospheric Limit

GOL with neutral collisions and neutral winds:

\[ 0 = E + u \times B - \frac{1}{en} J \times B - \frac{m_e}{e^2 n} \left( \nu_{ei} + \nu_{en} + \frac{m_e}{m_i} \nu_{in} \right) J + en \left( \nu_{en} - \nu_{in} \right) (u - u_n) \]

Steady state momentum equation:

\[ 0 = J \times B + \rho g - \nabla p - \nu_{in} (u - u_n) \]

\[ u = u_n + \frac{1}{\nu_{in}} \left[ J \times B + \rho g - \nabla p \right] \]

Substitute for \( u \) in GOL

\[ 0 = E + u_n \times B + \frac{1}{\nu_{in}} \left[ J \times B + \rho g - \nabla p \right] \times B \]

\[ - \frac{1}{en} J \times B - \frac{m_e}{e^2 n} \left( \nu_{ei} + \nu_{en} + \frac{m_e}{m_i} \nu_{in} \right) J \]

\[ + en \frac{\nu_{en} - \nu_{in}}{\nu_{in}} [J \times B + \rho g - \nabla p] \]

\[ J = \sigma \cdot [E + u_n \times B] + D \cdot \nabla p + \Gamma \cdot g \]
Transonic MHD Behavior of the Ionosphere

Eugene Dao
Ph.D. Dissertation
Cornell, 2013

(a) Simulation setup for a bubble at the equator.

(b) $E_{x1}$ for a bubble at the equator.

(c) $B_{z1}$ for a bubble at the equator.
Convection with IMF $B_z$ South (Dungey Cycle)
High-Latitude Ionospheric Convection
Current Systems

Force Balance:

\[ J_\perp = -\frac{1}{B^2} \nabla p \times B \]

\[ \nabla \cdot J = 0: \]

\[ J_\parallel = \int \nabla \cdot J_\perp \, ds \]

\[ J_\parallel = -\frac{B_{eq}}{B_{eq}^2} \cdot \nabla p_{eq} \times \nabla V \]

\[ V = \int_{\text{iono}}^{\text{eq}} \frac{ds}{B} \]
Energy Transport and Poynting’s Theorem

Poynting’s Theorem:

\[
\frac{\partial}{\partial t} \left[ \frac{\epsilon_0 |E|^2}{2} + \frac{|B|^2}{2\mu_0} \right] + \nabla \cdot \left[ \frac{E \times B}{\mu_0} \right] = -J \cdot E
\]

Energy Density

Energy Flux

Joule Heating

Ionospheric Joule Heating: Use E field in the neutral wind frame

\[
J \cdot E' = (\sigma \cdot E') \cdot E' = \sigma_P |E + u_n \times B|^2 = n_i m_i \nu_{in} |u_i - u_n|^2
\]

See Appendix A of Thayer and Semeter, 2004, JASTP.
Alfvén Wave Transmission Lines

**Electrostatic:**

\[
\mu_0 \frac{E}{\delta B} = \frac{1}{\Sigma_P}
\]

**Electromagnetic:**

\[
\mu_0 \frac{E}{\delta B} = \mu_0 v_A = \frac{1}{\Sigma_A}
\]

Reflected Alfvén wave forms such that

\[
\frac{E_{\text{down}} + E_{\text{up}}}{\delta B_{\text{down}} - \delta B_{\text{up}}} = \frac{1}{\Sigma_P}
\]

Reflection coefficient:

\[
\frac{E_{\text{up}}}{E_{\text{down}}} = \frac{\Sigma_A - \Sigma_P}{\Sigma_A + \Sigma_P}
\]

This simple transmission line model assumes

- Ionosphere is thin slab
- Alfvén speed above ionosphere is constant
Effects of Conductance Distributions (Lotko et al. 2014)

Uniform

Causal - empirical

Hall depletion

Polar ionosphere

Equatorial magnetosphere

R. H. Varney (SRI)

Geospace Electrodynamics

June 20, 2016
The Magnetosphere-Ionosphere “Gap” Region

- Magnetosphere models operate outside of $2 - 3 \, R_E$
- Ionosphere-thermosphere models operate up to $\sim 600 \, \text{km}$ altitude ($1.1 \, R_E$)
- Electrostatic fields assumed to map along field lines in between
Ambipolar Electric Fields

\[ E + u \times B - \frac{1}{en} J \times B = -\frac{1}{en} \nabla p_e \]

\[ E_{\parallel} = -\frac{1}{en} \nabla_{\parallel} p_e \]
Classical Polar Wind

- In steady state ambipolar field balances gravity for major ion species ($\text{O}^+$)

- Light minor ions ($\text{H}^+$ and $\text{He}^+$) feel same field

$$eE_\parallel = m_{\text{O}^+} + g$$

$$m_{\text{H}^+} + g$$

$$m_{\text{O}^+} + g$$
See Wilson et al. (1997), Kitamura et al. (2012, 2013), and Varney et al. (2014).
The Knight Relation and Mono-energetic Aurora

How can field lines carry upwards FAC?

\[ \mathbf{F} = -\frac{mv^2}{2B} \nabla B \]

"Ambipolar Term" with anisotropy

\[ -en\mathbf{E} = \nabla \cdot \mathbf{P}_e \]

\[ = \nabla \cdot \left[ p_\parallel \hat{b}\hat{b} + p_\perp (\mathbf{I} - \hat{b}\hat{b}) \right] \]

\[ = \nabla p_\parallel + (\mathbf{I} - \hat{b}\hat{b}) \cdot \nabla (p_\perp - p_\parallel) \]

\[ - (p_\perp - p_\parallel) (\mathbf{I} - 2\hat{b}\hat{b}) \cdot \frac{1}{B} \nabla B \]

\[ -en\hat{b} \cdot \mathbf{E} = \hat{b} \cdot \nabla p_\parallel + (p_\perp - p_\parallel) \hat{b} \cdot \frac{1}{B} \nabla B \]

Fields required to overcome the mirror-force term can produce \( > 1 \) kV potential drops!
Energetic Ion Outflow

How do heavy ions escape gravity?

- Parallel electric fields
- Transverse acceleration combined with mirror force lifting

[Diagram of energetic ion outflow]

Image courtesy of the ePOP team

Ion conic (Bouhram et al. 2004)

Akebono/LEP

\[ r = 2.5 \, R_E \]
Ion Outflow as a Multistep Process

Strangeway et al. (2005)

Electrostatics Inner Mag MHD SWMI Coupling Polar Wind

R. H. Varney (SRI) Geospace Electrodynamics June 20, 2016 44 / 45
Some Open Research Areas

- Collisionless Reconnection
- Conjugacy
- Inductive Coupling
- Tail-Inner Mag. Interactions
- Particle Acceleration
- Ion Outflow Effects on Magnetotail